AN ECONOMIC RETENTION MODEL

FOR

EXCESS NAVY MATERIAL

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ABSTRACT

This study evaluates alternative UICP (Uniform Inventory Control Program) Navy Economic Retention models. The current Navy Economic Retention Model was developed in 1965 for consumables only and was restricted in precision by computer constraints and simplifying assumptions. A replacement model is proposed that applies to RFI (Ready-for-Issue) consumable and repairable assets, as well as NRFI (Not-Ready-for-Issue) repairable assets. The proposed model represents an improved mathematical formulation that takes advantage of current ADP (Automatic Data Processing) capabilities and, thus, eliminates many simplifying assumptions of the current model. The proposed model, under current constraints, computed a lower economic retention requirement for the total of all Navy items. However, implementation of the proposed model based solely on economic criteria would increase the economic retention quantity.

EXECUTIVE SUMMARY

1. Background. Variations in Fleet programs and supply policy create apparent material excesses over time from one item of supply to the next. In the mid-60s, an economic retention requirements model was developed to assist in the disposal decision for consumable items. The model was initially constrained to establish limits to any decision and, to permit efficient operation, the model employed simplifying assumptions and approximations. Over time the limits have been modified and the model has been extended to use on repairable items. A preliminary analysis of the current model indicates need for improved management of both consumable and repairable disposal recommendations.

2. Objective. To develop an improved Economic Retention model for Navy consumable and repairable assets in long supply. To apply more advanced model solution techniques consistent with improved computer hardware and software capabilities.

3. Approach. The current model and assumptions were examined to note parameters considered, those omitted, use of overrides and other limits, the technique used to approximate the theoretical problem solution, and methods of application in the stratification and disposal decision. A new model was fashioned to consider both consumable and ready-for-issue repairables, while a second algorithm was developed to consider not-ready-for-issue material assets. Proven mathematical techniques were applied to obtain exact problem solutions.

4. Findings. The proposed model was developed without need for simplifying assumptions and, as a consequence, determines the economic retention requirement more precisely. The models incorporate new parameters as well as using many of those in the current model to give a more accurate problem solution. A mathematical routine, the modified binary technique, an iterative procedure, provides exact problem solutions. The feasibility of models developed was demonstrated through sensitivity analysis. The practicality of the models was demonstrated using UICP stratification data. The results of the model application indicate the new model using old constraints retain fewer assets than the current model. The new model with new constraints retain substantially greater assets than the current model as constrained.

5. Conclusions. The objectives of the study were accomplished. An improved economic retention model was developed. Improved mathematical features were introduced to give more accurate treatment of the decision variables consistent with improved hardware and software capabilities. There remains the need to precisely establish model parameter values (beyond the scope of this study) and establish procedures to maintain these parameters. The models are adaptable to current versions of UICP stratification and disposal applications.

GLOSSARY OF TERMS AND SYMBOLS

1. a - obsolescence risk rate.

2. A_1 - procurement order cost.

3. A_2 - manufacturer's set-up cost.

4. A_3 - repair administrative cost.

5. A_4 - repair set-up cost.

6. A condition - assets that are in A condition are ready-for-issue off the shelf to the customer with no rework or repair required.

7. Apportionment year - covers the 12 months following the Current Year or the remainder of the fiscal year after the processing of stratification when the Current Year is not computed.

8. Budget year - the stratification horizon which extends from the end of the Apportionment Year to the following 30 September. It is always four quarters in length and covers the period for which the budget is being prepared.

9. β - administrative cost to dispose of a unit of stock.

10. c - standard unit price.

11. Current year - covers the remaining two quarters of the fiscal year after the 31 March stratification processing date.

12. d - probability of inventory loss in storage (includes pilferage).

13. D_g - annual demand forecast for two years after budget year.

14. Economic retention requirement - a value determined by considering recurring demand, obsolescence rate, shelf life, order quantity and order cost. From the calculation it can be decided whether excess stock should be kept by determining if the cost to retain the material is less than the cost to reprocure the material at a later date.

15. ERR - economic retention quantity determined in current retention model with current constraints.

16. ERR_1 - economic retention quantity determined in proposed retention model with current constraint

17. ERR_2 - economic retention quantity determined in proposed retention model with new constraints.

18. i - discount rate.

19. j - disposal return rate for not-ready-for-issue material.

20. k - repair price.

21. m - time in years.

22. MARK - classification of items in inventory based on item characteristics.

    MARK 0 - insurance items with quarterly demand forecast below 0.25 units.

    MARK I - low demand, low cost items with quarterly demand forecast between 0.25 units and 5.00 units and standard price below $50.00.

    MARK II - high demand, low sales, and low cost items with quarterly forecast of demand above 5.00 units and quarterly sales below $75.00

    MARK III - low demand high cost items with quarterly demand forecast between 0.25 units and 5.00 units and standard price above $50.00.

    MARK IV - high demand, high sales with forecasted quarterly demand greater than 5.00 units and quarterly sales greater than $75.00

23. NRFI - not-ready-for-issue material.

24. p - disposal return rate for ready-for-issue material.

25. PER - sum of all assets stratified to all requirements now.

26. Q_1 - basic order quantity.

27. $\hat{Q}_{1b}$ - constrained order quantity for budget year.

28. $\hat{Q}_{2b}$ - constrained repair quantity for budget year.

29. QER - sum of initial constrained retention limit, opening position backorders, and planned program requirements.

30. RD3 - recurring demand during the budget year.

31. RFI - ready-for-issue material.

32. s - storage cost.

33. S - shelf life.

34. SER - sum of all requirements considered before the economic retention requirement.

35. SSOH - serviceable stock on-hand in A condition.

36. t - maximum number of years of annual demand for economical retention under current model.

37. t_1 - optimum number of years of annual demand for economical retention under proposed model for ready-for-issue material.

38. t_2 - optimum number of years of annual demand for economical retention of not-ready-for-issue material under proposed model.

39. T_1 - transportation cost for disposal of material.

40. T_2 - transportation cost to move carcass to designated overhaul point.

41. TA1 - total assets, opening position.

42. W_1 - initial constrained retention limit.

43. X_1 - basic reorder level.

44. Y - units per item to be held in retention by model.

I. INTRODUCTION

The Navy Economic Retention Model was developed in 1965 for application to consumable items. The model was designed to assist inventory managers in determining an apparent economic quantity of material to retain for potential future use. Reference (a) of APPENDIX A documents the original version of the model. Several simplifying assumptions and mathematical approximations were necessary to obtain efficient operation of the model on the then available automatic data processing equipment. Since initial development, several modifications have been made to the model to implement policy changes and for application to repairable items. Constraints and overrides have been applied over the years to implement change to management philosophy or for financial necessity.

Many factors tend to cause fluctuation in demand forecasts for repair parts which have a direct impact on the determination of excess (or apparent excess) stocks. Consider the following hypothetical example. Twenty ships have one application of the same equipment and one of the repair parts must be replaced annually. The annual demand is 20 units per year and if the supply system has 40 units of this part in stock, then two years worth of stock are on-hand. Now suppose that five of the ships have the equipment replaced and 10 of the ships are deactivated. The system stock of 40 units would now equate to eight years worth of support stock which creates an excess situation. The inventory manager must decide what portion of the on-hand stock is to be declared excess. Often the determination is made when assets are stratified in UICP (Uniform Inventory Control Program) as a part of the budget formulation. The more precise the identification and consideration of factors impacting the decision, the more optimal (economic) the decision. The original model was designed expressly for consumables. A preliminary analysis of the model (as modified) indicates possibility for improved management of both consumable and repairable items. Factors relevant to repairable item management; i.e., carcass transportation costs, should be introduced. The original assumptions and approximations should be reduced to give a more exact answer, and new techniques should be examined for potential increase in accurate and efficient processing. Reference (b) of APPENDIX A describes the tasking for improving the Navy Economic Retention Model formulation.

II. MODEL DEVELOPMENT

Textbook solutions rarely solve operational problems directly and completely. The theory behind the solution is usually demonstrated on a hypothetical situation where but few variables are recognized. Judgment and experience are required for successful application of such algorithms because assumptions are required to fashion the operational situation to the algorithm or model. Judgment is also required to formulate the relationship of the various factors in the operational situation and to assign meaningful values to constraints and variables used in the model. This section of the study compares assumptions, the formulation, constraints and variables of the original and proposed models. The solution techniques inherent in the model are also discussed:

A. ASSUMPTIONS.

1. Original Model.

a. The holding cost of material is independent of the event of obsolescence.

b. The total holding cost of material at the end of t years is t times the annual holding cost rate.

c. The time value of money concept does not affect holding costs in the year incurred but only after t years.

d. The annual probability of obsolescence from year to year is independent.

e. The obsolescence rate represents the probability of an item becoming technologically and instantaneously obsolete in any given year.

f. The standard price charged on a procurement action in the future will be at the standard price in the file today.

2. Proposed Model.

a. The holding costs for material will be incurred only if an item is not obsolete. Obsolete item assets will be disposed immediately.

b. The holding costs are incurred annually over time and are subject to valuation by time value of money concept.

c. The annual probability of obsolescence is uniformly (linearly) distributed over the expected life of an item.

d. The obsolescence rate represents the probability of an item becoming technologically and instantaneously obsolete in any given year.

e. The standard price charged on a procurement action in the future will be at the standard price in the file today.

f. The probability of an item being lost in storage is independent from one year to the next.

g. The probability of item obsolescence is independent of the probability of loss in storage.

3. Comments on Assumptions. Items which are obsolete have zero present and zero anticipated future usage. It seems reasonable to assume obsolete items will be disposed eliminating holding costs. Costs to maintain and operate warehouses indicate annual holding costs occur. The occurrence of these costs at specific points in time makes the use of the value of money (discounting) concepts appropriate. The obsolescence and discounting factors change in relative value from year to year. Consequently, an annual holding cost cannot simply be multiplied by t years to compute the total holding costs over t years.

A basic assumption is the uniform distribution of annual obsolescence rate over the life of the item. An item with a 20 year life would have an annual obsolescence of .05 for each and every year. FIGURE I illustrates the principle. The assumption is considered reasonable because knowledge of the individual item's exact distribution of obsolescence is unknown. The distribution would be most difficult to establish empirically due to the uniqueness of obsolescence of each item.

UNIFORM DISTRIBUTION OF ANNUAL OBSOLESCENCE
AND CUMULATIVE DISTRIBUTION OF ANNUAL OBSOLESCENCE

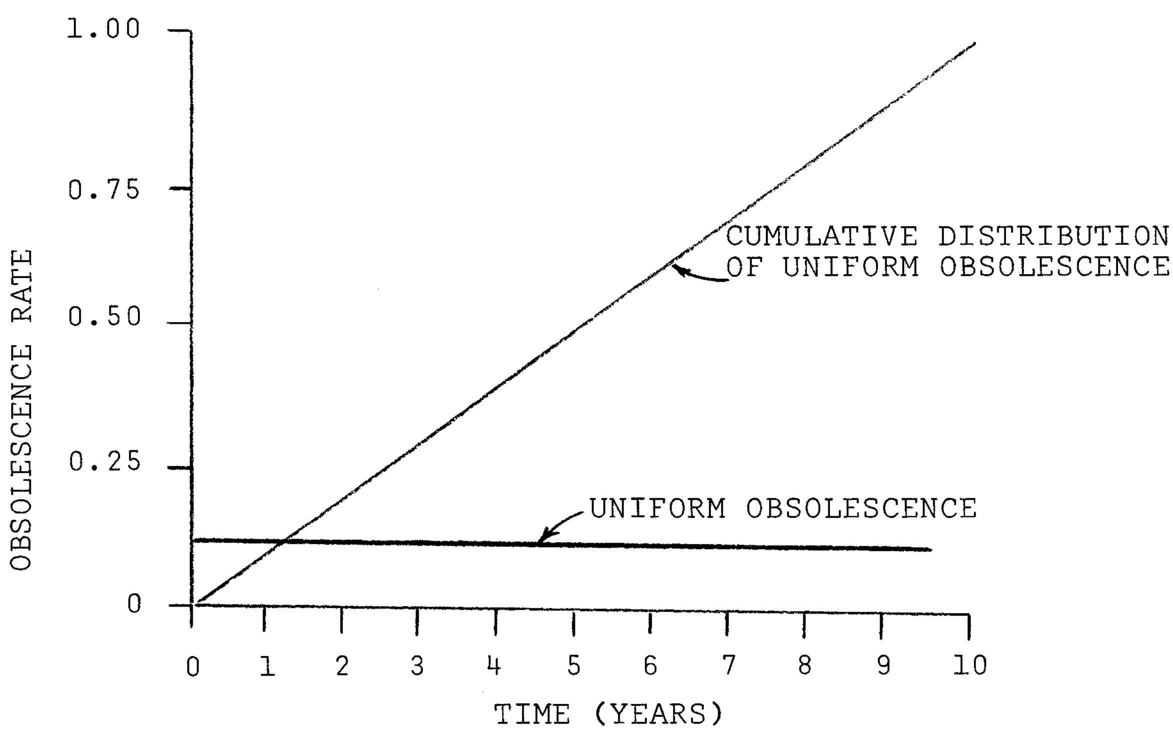

FIGURE I

B. <u>THE MODEL</u>. Both models, the original and proposed, use several common variables. The symbols and titles of the variables are shown in the glossary.

Excess material should be held only where economic criteria indicate the costs of reprocurement at some future time will exceed the costs to hold the material. Holding costs include the opportunity cost of not liquidating the assets through disposal, repair costs, and physical storage costs. Material should be disposed if:

$$\text{Proceeds for disposal} + \text{repair costs} + \text{storage costs} > \text{reprocurement costs}$$

FIGURE 2 demonstrates the relationship graphically. The example indicates that 6.3 years of demand, based on present forecast, should be held.

MATERIAL RETENTION VS DISPOSAL

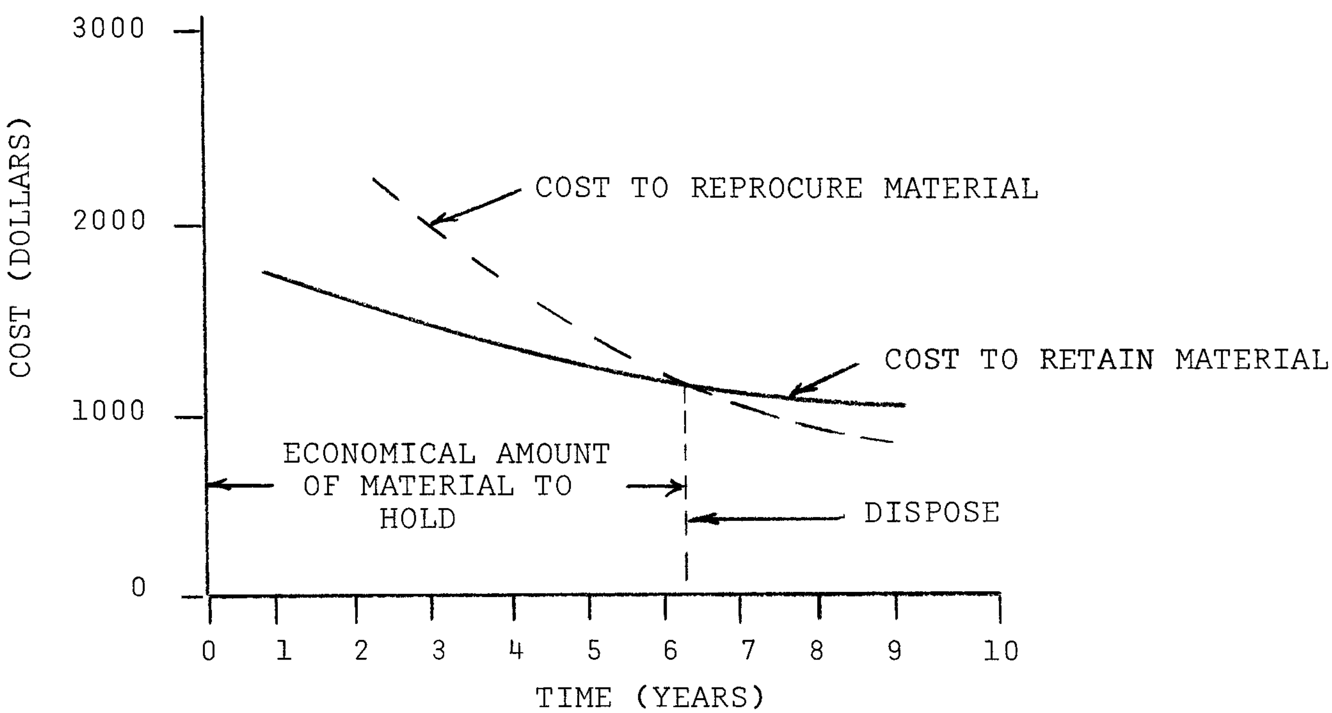

FIGURE 2

1. Current Model Formulation. Reference (a) of APPENDIX A contains the formula and rationale for the Navy Economic Retention Model. The model is predicated on the assumptions cited earlier. The basic equation is:

$$pc + \frac{sct}{(1 + i)^t} > \frac{(1 - a)^t}{(1 + i)^t}\left(c + \frac{A_1 + A_2}{\hat{Q}_{1b}}\right) \qquad (1)$$

In the equation $t = \frac{Y}{D_g}$ and represents the number of years of assets to be held. For example, if Y = 1000 units and D_g = 500 units per year, then t = two years of demand. The first term in the equation represents the proceeds for immediate disposal. If the material is held, this term represents the opportunity cost for not liquidating the assets. The second term represents the physical costs associated with holding material for t years, where t is the solution variable of the model. The discount factor $\left(\frac{1}{1 + i}\right)$ is for year t only and obsolescence rate (a) is not considered as a factor of holding cost. The last term represents the costs for reprocuring disposed material. The factor $(1 - a)^t$ indicates independence of annual probabilities of obsolescence.

The model was solved by approximating $(1 - a)^t$ with $(1 - ta)$, which is accurate when ta is a small value. The larger the product of t and a, the less accurate the approximation. Similarly, $(1 + i)^t$ is approximated by $(1 + ti)$. Substituting in equation (1):

$$pc + \frac{sct}{(1 + ti)} > \frac{(1 - ta)}{(1 + ti)}\left(c + \frac{A_1 + A_2}{\hat{Q}_{1b}}\right) \tag{2}$$

or $$pc(1 + ti) + sct > (1 - ta)\left(c + \frac{A_1 + A_2}{\hat{Q}_{1b}}\right)$$

or $$pc + pcti + sct > (1 - ta)\left(c + \frac{A_1 + A_2}{\hat{Q}_{1b}}\right)$$

The breakeven point occurs when holding costs equal reprocurement costs which allows the > sign to be replaced by the = sign.

To rearrange the equation to solve for t:

$$pc + pcti + sct = (1 - ta)\left(c + \frac{A_1 + A_2}{\hat{Q}_{1b}}\right)$$

$$pc + pcti + sct = c + \left(\frac{A_1 + A_2}{\hat{Q}_{1b}}\right) - c\ ta - ta\left(\frac{A_1 + A_2}{\hat{Q}_{1b}}\right)$$

$$pcti + sct + c\ ta + ta\left(\frac{A_1 + A_2}{\hat{Q}_{1b}}\right) = c - pc + \left(\frac{A_1 + A_2}{\hat{Q}_{1b}}\right)$$

$$t\left[pci + sc + ca + a\left(\frac{A_1 + A_2}{\hat{Q}_{1b}}\right)\right] = c - pc + \left(\frac{A_1 + A_2}{\hat{Q}_{1b}}\right)$$

$$t\left[pi + s + a\left(1 + \left(\frac{A_1 + A_2}{c\ \hat{Q}_{1b}}\right)\right)\right] = 1 - p + \left(\frac{A_1 + A_2}{c\ \hat{Q}_{1b}}\right)$$

$$t = \frac{1 - p + \left(\dfrac{A_1 + A_2}{c\ \hat{Q}_{1b}}\right)}{pi + s + a\left(1 + \left(\dfrac{A_1 + A_2}{c\ \hat{Q}_{1b}}\right)\right)} \tag{3}$$

To obtain an exact solution to equation (1), iterative techniques would be required. The data processing equipment available when the original model was developed would have been inefficient in obtaining the exact solution and the equipment was required for many other determinations of equal or greater priority. Equation (3) was used to get an approximate answer with a single iteration.

2. Proposed Model Formulation. Two expressions are developed for the proposed model. The first is comparable to the original model and applies to consumable and RFI (Ready-for-Issue) material. The second considers the uniqueness of NRFI (Not-Ready-for-Issue) material and introduces new variables. The symbols are defined in the glossary.

a. Consumable/RFI Retention Quantity. The proceeds from immediate disposal of material may be expressed as pc. The net proceeds are determined by considering the administrative and transportation costs incurred in the disposal process. A representation for net proceeds is:

$$pc - \frac{\beta}{t_1 D_g} - T_1$$

The annual storage costs can be expressed by:

$$sc(1 - t_1 a)(1 - d)^{t_1}$$

The value of t_1 is the optimal years of consumable/RFI material to retain and is the solution variable of the model. The factor $(1 - t_1 a)$ is considered to be appropriate because storage costs occur only before material becomes obsolete and the distribution of obsolescence is assumed uniform over time. A linear cumulative distribution of nonobsolescence is also assumed. The factor $(1 - d)^{t_1}$ is considered appropriate because the probabilities of the incidents of loss are considered to be independent from year to year and independent of the probability of obsolescence. Here $t_1 = Y/D_g$ is the time to hold the Y^{th} unit of stock (number of years of stock based on demand forecast). The discounted total holding cost for the proposed model is:

$$pc - \frac{\beta}{t_1 D_g} - T_1 + sc\ (1 - a)\left(\frac{1 - d}{1 + i}\right) + sc\ (1 - 2a)\left(\frac{1 - d}{1 + i}\right)^2 + \ldots + sc\ (1 - t_1 a)\left(\frac{1 - d}{1 + i}\right)^{t_1}$$

or

$$pc - \frac{\beta}{t_1 D_g} - T_1 + sc \sum_{m=1}^{t_1} \left[(1 - ma)\left(\frac{1 - d}{1 + i}\right)^m\right]$$

The discounted expected cost to reprocure, in year t_1, a unit of material disposed now, can be expressed as:

$$\left[\frac{1 - t_1 a}{(1 + i)^{t_1}}\right]\left[c + \left(\frac{A_1 + A_2}{\hat{Q}_{1b}}\right)\right]$$

Using the same rationale of the original model, material should be disposed when the following relationship exists:

$$pc - \frac{\beta}{t_1 D_g} - T_1 + sc \sum_{m=1}^{t_1} \left[(1 - ma)\left(\frac{1 - d}{1 + i}\right)^m\right] > \left[\frac{1 - t_1 a}{(1 + i)^{t_1}}\right]\left[c + \left(\frac{A_1 + A_2}{\hat{Q}_{1b}}\right)\right] \quad (4)$$

The optimal level of stock that should be held is t_1 years of demand and occurs when the > sign is replaced with the equals sign in equation (4).

The term $sc \sum_{m=1}^{t_1} \left[(1 - ma)\left(\frac{1 - d}{1 + i}\right)^m\right]$ produces only discrete values of t_1, therefore, the optimal value of t_1 can not be found as a fraction of a year. If the disposal function is to be a continuous function rather than discrete, the formulation can be modified to give a continuous (fractional) value for t_1. The following applies:

$$sc \int_{m=0}^{t_1} \left[(1 - ma)\left(\frac{1 - d}{1 + i}\right)^m\right] dm \approx sc \sum_{m=1}^{t_1} \left[(1 - ma)\left(\frac{1 - d}{1 + i}\right)^m\right]$$

The integral can be integrated by parts:

$$sc \int_{m=0}^{t} \left[(1 - ma)\left(\frac{1 - d}{1 + i}\right)^m\right] dm =$$

$$sc \left\{ \frac{\left(\frac{1 - d}{1 + i}\right)^{t_1} \left[(1 - t_1 a)\ln\left(\frac{1 - d}{1 + i}\right) + a\right] - \left[\ln\left(\frac{1 - d}{1 + i}\right) + a\right]}{\left[\ln\left(\frac{1 - d}{1 + i}\right)\right]^2} \right\}$$

Substituting into equation (4) with the equals sign:

$$pc - \frac{\beta}{t_1 D_g} - T_1 + sc\left\{ \frac{\left(\frac{1 - d}{1 + i}\right)^{t_1}\left[(1 - t_1 a)\ln\left(\frac{1 - d}{1 + i}\right) + a\right]}{\left[\ln\left(\frac{1 - d}{1 + i}\right)\right]^2} \right\} = \left(\frac{1 - t_1 a}{(1 + i)^{t_1}}\right) \left[c + \left(\frac{A_1 + A_2}{\hat{Q}_{1b}}\right)\right]$$

Subsequent analysis (see TABLE I, page 31) showed that β was insensitive and had minimal impact on the solution of the model, therefore should be dropped from the formulation which now becomes:

$$pc - T_1 + sc\left\{\frac{\left(\frac{1-d}{1+i}\right)^{t_1}\left[(1 - t_1 a)\ell n\left(\frac{1-d}{1+i}\right) + a\right] - \left[\ell n\left(\frac{1-d}{1+i}\right) + a\right]}{\left[\ell n\left(\frac{1-d}{1+i}\right)\right]^2}\right\} =$$

$$\left(\frac{1 - t_1 a}{(1 + i)^{t_1}}\right)\left[c + \left(\frac{A_1 + A_2}{\hat{Q}_{1b}}\right)\right] \tag{5}$$

b. NRFI Retention Quantity. The logic used for the RFI determination and that for the NRFI determination is analogous. To differentiate, t_2 will be the symbol used for the solution variable and j will represent the rate of return for carcass disposal. Several new variables will be introduced, all of which are shown in the glossary. New variables include added transportation and administration costs, and the actual costs to repair. The net proceeds for disposal of a unit of stock can be expressed mathematically:

$$jc - \frac{\beta}{t_2 D_g} - T_1$$

Annual storage costs may be expressed:

$$sc \sum_{m=1}^{t_2} \left[(1 - ma) \left(\frac{1 - d}{1 + i} \right)^m \right]$$

And unit administrative repair costs:

$$\frac{A_3 + A_4}{\hat{Q}_{2b}}$$

Actual repair and repair transportation costs:

$$k + T_2$$

Then the total discounted expected repair cost for the Y^{th} unit is:

$$(1 - t_2 a) \left(\frac{1 - d}{1 + i} \right)^{t_2} \left(\frac{A_3 + A_4}{\hat{Q}_{2b}} + k + T_2 \right)$$

The cost to procure a replacement unit for one disposed now t_2 years later is expressed as:

$$\left[\frac{1 - t_2 a}{(1 + i)^{t_2}} \right] \left[c + \frac{A_1 + A_2}{\hat{Q}_{1b}} \right]$$

The optimal level of NRFI stock to retain (on an economic basis) is the demand for t_2 years, where:

$$jc - \frac{\beta}{t_2 D_g} - T_1 + sc \sum_{m=1}^{t_2} \left[(1 - ma)\left(\frac{1 - d}{1 + i}\right)^m \right] + \left(\frac{A_3 + A_4}{\hat{Q}_{2b}} + k + T_2\right)\left[(1 - t_2 a)\left(\frac{1 - d}{1 + i}\right)^{t_2}\right]$$

$$= \left[\frac{1 - t_2 a}{(1 + i)^{t_2}}\right]\left[c + \frac{A_1 + A_2}{\hat{Q}_{1b}}\right] \tag{6}$$

Assuming disposal is a continuous function and recognizing β is insignificant (as shown earlier), then equation (6) becomes:

$$jc - T_1 + sc\left\{\frac{\left(\frac{1 - d}{1 + i}\right)^{t_2}\left[(1 - t_2 a)\ell n\left(\frac{1 - d}{1 + i}\right) + a\right] - \left[\ell n\left(\frac{1 - d}{1 + i}\right) + a\right]}{\left[\ell n\left(\frac{1 - d}{1 + i}\right)\right]^2}\right.$$

$$+\left(\frac{A_3 + A_4}{\hat{Q}_{2b}} + k + T_2\right)(1 - t_2 a)\left(\frac{1 - d}{1 + i}\right)^{t_2} = \left[\frac{1 - t_2 a}{(1 + i)^{t_2}}\right]\left[c + \frac{A_1 + A_2}{\hat{Q}_{1b}}\right] \tag{7}$$

Equations (5) and (7) use the iterative approach for determination of values for t_1 and t_2 rather than reduce the equations using approximations to give the value of the solution variable in a single iteration as does the current model. A sample solution is provided later.

C. <u>SOLUTION TECHNIQUES</u>. Solution of equations (5) and (7) requires sophisticated mathematical techniques to attain accurate answers efficiently. Reference (c) of APPENDIX A describes an iterative solution method known as linear interpolation (secant method).

The following relationships pertain in this technique:

$$t_{n+1} = \frac{1}{f(t_n) - f(t_{n-1})} \begin{vmatrix} t_{n-1} & f(t_{n-1}) \\ t_n & f(t_n) \end{vmatrix} = \frac{(t_{n-1})(f(t_n)) - (t_n)(f(t_{n-1}))}{f(t_n) - f(t_{n-1})} \qquad (8)$$

to find successive trial values for t_1 or t_2. The solution value for t_1 is found when equation (5) is transformed to:

$$pc - T_1 + sc\left\{\frac{\left(\frac{1-d}{1+i}\right)^{t_1}\left[(1 - t_1 a)\ell n\left(\frac{1-d}{1+i}\right) + a\right] - \left[\ell n\left(\frac{1-d}{1+i}\right) + a\right]}{\left[\ell n\left(\frac{1-d}{1+i}\right)\right]^2}\right\}$$

$$- \left(\frac{1 - t_1 a}{(1+i)^{t_1}}\right)\left[c + \frac{A_1 + A_2}{\hat{Q}_{1b}}\right] = 0 \qquad (9)$$

The solution value to equation (7) is found similarly:

$$jc - T_1 + sc\left\{\frac{\left(\frac{1-d}{1+i}\right)^{t_2}\left[(1 - t_2 a)\ell n\left(\frac{1-d}{1+i}\right) + a\right] - \left[\ell n\left(\frac{1-d}{1+i}\right) + a\right]}{\left[\ell n\left(\frac{1-d}{1+i}\right)\right]^2}\right\}$$

$$+ \left(\frac{A_3 + A_4}{\hat{Q}_{2b}} + k + T_2\right)(1 - t_2 a)\left(\frac{1-d}{1+i}\right)^{t_2} - \left(\frac{1 - t_2 a}{(1+i)^{t_2}}\right)\left[c + \frac{A_1 + A_2}{\hat{Q}_{1b}}\right] = 0 \qquad (10)$$

Another method to solve such equations is known as the modified binary technique. This method begins with the development of a trial solution using a value of t equal to one. Each iteration increases the value by one until the direction of the model inequality changes which indicates the solution lies between the last integer and the previous one tried. A binary search technique is then used to find the exact value of t by continually bisecting the differences between the last trial value and the previous until the solution is found. The following example is offered to illustrate:

Assume: $s = .01$ $d = .04$ $p = .05$ $A_1 = 2000$ $Q_1 = 50$

$a = .10$ $i = .10$ $T_1 = .05$ $A_2 = 2000$ $c = 2000$

Consider the model:

$$pc - T_1 + sc\left\{\frac{\left(\frac{1-d}{1+i}\right)^{t_1}\left[(1-t_1 a)\ell n\left(\frac{1-d}{1+i}\right) + a\right] - \left[\ell n\left(\frac{1-d}{1+i}\right) + a\right]}{\left[\ell n\left(\frac{1-d}{1+i}\right)\right]^2}\right\}$$

$$< \left(\frac{1 - t_1 a}{(1+i)^{t_1}}\right)\left[c + \frac{A_1 + A_2}{Q_1}\right]$$

t_1	Left Side of Equation	Equality	Right Side of Equation
1	117.73	<	1701.81
2	131.62	<	1375.20
3	142.32	<	1093.91
4	150.41	<	852.40
5	156.39	<	645.75
6	160.66	<	469.64
7	163.56	<	320.21
8	165.37	<	194.06
9	166.32	>	88.21
8.5	165.94	>	138.77
8.25	165.81	=	165.81

The direction of the inequality changed between values for t_1 of 8 and 9. Bisecting the interval twice provided the exact solution to the equation.

For test purposes, both techniques were programmed in FORTRAN IV and applied to UICP data. The method of linear interpolation proved unsuitable due to a tendency of nonconvergence. The binary search method converged without exception as shown in the above example. The FORTRAN IV routine incorporating the binary search method can be converted for incorporation into UICP. The modified binary search technique is simple to apply and easy to comprehend. The values for the solution variable will be more accurate than the current model values.

D. <u>CONSTRAINTS</u>. The variety and number of items in the Navy inventory dictate against allowing the model (or any set of rules) to run unfettered. Operational factors such as shelf life make model constraints an economic necessity. The initial constrained retention limit, W_1, is computed as follows:

1. <u>Current Model</u>.

   a. MARK I or II

   $$W_1 = [\min (D_g S;\ D_g/a) + 0.999]^+ \tag{11}$$

   b. MARK III or IV

   $$W_1 = [\min (D_g t;\ D_g S) + 0.999]^+ \tag{12}$$

   c. Repairables with regenerations $\geq$ demand

   $$W_1 = [4D_g + 0.5]^+ \tag{13}$$

   d. Repairables with regenerations < demand

   $$W_1 = [D_g t + 0.999]^+ \tag{14}$$

W_1 represents the total assets to be held after considering economic criteria: shelf life, obsolescence, gross system demands at end of leadtime, and RFI regenerations at end of leadtime. Equations (11) and (13) reflect policies that limit the application of economic criteria in decision making. The rationale apparent in equation (11) is that due to the relative inexpensive items, economic considerations are not appropriate in the hold/dispose decision and the economic solution variable t_1 is not included. Equation (11) says hold the smaller of expected demand over shelf life or the demand expected during the life of the item. Equation (13) says to hold four years of stock based on the forecast of demand two years after the budget year. The rationale seems to be that when regenerations exceed demand, zero attrition occurs and procurement is not needed to meet forecasted demand. In equations (12) and (14), the solution variable is found for use in computing W_1. This is intuitively appealing because in equation (12) the expensive, high sales items are included. In equation (14), procurements will be required to meet forecasted demand and the economics of hold/dispose are appropriate considerations.

2. Current Model Final Constraints. Optimal financial determinations are not the sole considerations in the hold/dispose problem. Planned program changes, for example, may tend to alter the optimal financial answer. The following equations reflect policy for various conditions, much of which is based on historical occurrences or expectations based on judgment.

a. MARK 0

$$\text{ERR} = 2Q_1 + X_1 + \text{scheduled funded planned requirements for the second year after the budget year} + \text{other planned requirements} \quad (15)$$

b. Provisioned Items

$$\text{ERR} = 0 \quad (16)$$

c. Repairable with RFI $\geq W_1 + \text{QER}$

$$\text{ERR} = \text{Max}(0; \text{QER}-\text{SER}) \quad (17)$$

d. Repairable with RFI > SER but < QER

$$\text{ERR} = \text{Max}[0; \min(\text{QER} - \text{SER}; \text{RFI} + 2\text{RD3} - \text{SER})] \quad (18)$$

where RD3 = recurring demand during budget year

e. Repairable item with RFI < SER

$$ERR = 2RD3 \tag{19}$$

f. All other

$$ERR = \text{Max}\ \{0;\ \min[TA1;\ \text{Max}\ (QER;\ SER)] - PER\} \tag{20}$$

3. Proposed Model Initial Constraints. The proposed model continues the policies for several categories of material as in the current model. The computation where repairables with regenerations less than demand is changed to give consideration to both solution variables t_1 and t_2, purchase and repair.

a. MARK I or II - see equation (11).

b. MARK III or IV - see equation (12), but substitute t_1 for t.

c. Repairables with regeneration $\geq$ demand - see equation (13).

d. Repairables with regenerations < demand -

$$W_1 = \left[\text{Min}(t_1 D_g;\ SSOH) + \text{Max}\ \left|0;\left(\frac{t_1}{t_2}\right)\ (t_1 D_g - SSOH) + .999\right|\right]^{+} \tag{21}$$

Equation (21) considers the optimal number of both RFI $[\text{Min}\ (t_1 D_g;\ SSOH)]$ and NRFI $\left[\text{Max}\ \left|0;\ \left(\frac{t_1}{t_2}\right)(t_1 D_g - SSOH)\right|\right]$ to hold.

4. Proposed Model Final Constraints. The proposed model again continues a position of the policy found in the current model.

a. MARK 0 - see equation (15).

b. Provisioned items - see equation (16).

c. All other - see equation (17).

E. <u>MODEL VARIABLES</u>. The proposed model uses the variables of the current model plus variables for the added economic considerations introduced. The values for variables pertinent to the model have several sources. DOD (Department of Defense) and NAVSUP (Naval Supply Systems Command) directives provide guidance on determination for some of the variables or the actual value. Historical data serves as the basis for a number of the variables. Large data bases provide accurate values for variables and data samples provide less accurate values. Other values are based on experience or assumed. Various methods are demonstrated for illustration.

1. Obsolescence rate, symbol a, may be computed by estimating the useful life of an item, in this study the probability of obsolescence is assumed to be uniformly distributed. The computation:

$$a = \frac{1}{\text{useful life of the item in years}}$$

Reference (d) of APPENDIX A provides policy for computing the obsolescence rate. The value for a is:

$$a = \frac{\text{transfers to all property disposal officers}}{\begin{array}{l}\text{stratified on-hand and on-order assets representing} \\ \text{the maximum expected on-hand and on-order quantities} \\ \text{at any point in time}\end{array}}$$

The latter method is considered impractical for estimating obsolescence rates for the current or proposed models due to data quality.

2. Discount rate, symbol i, is set by reference (d) of APPENDIX A to be 10% per year. The rate is predicated on the economic theory of investment opportunity cost.

3. Storage cost, symbol s, is established as 1% by reference (d) of APPENDIX A. This cost represents the cost to receive, stow and issue material, including maintenance of warehouses, etc.

4. Pilferage and other inventory losses, symbol d, are discussed in reference (d) of APPENDIX A, but guidance for computation is lacking. During reference (e) of APPENDIX A, a sample of data was obtained from which an estimation for d was determined, based on the assumption that the probability of loss of any unit of one item equals the probability of the loss of any unit of any other item.

CATEGORY	ITEMS EXPERIENCING LOSS	TOTAL ITEMS	PERCENT LOSS
APA	386	10,008	3.8
NSF	615	6,641	9.2
TOTAL	1,001	16,649	6.0

The estimated value for d based on the given sample is 6%.

5. The procurement order cost, symbol A_1, is estimated in accordance with reference (d) of APPENDIX A. Values used in UICP at the time of the study were found to vary by ICP (Inventory Control Point) and by type of procurement action as follows:

DEN NR	ICP	VALUE
V015 - Order Cost (MARK I and II)	SPCC ASO	$ 70.00 108.27
V041 - Order Cost (Low Value Demand)	SPCC ASO	102.00 108.27
V042 - Negotiated Procurement	SPCC ASO	275.00 183.54
V043 - Advertised Procurement	SPCC ASO	326.00 183.54

6. The local transportation cost, symbol T_1, is peculiar to the proposed model and is not specified in known official directive. With the assistance of NSC Charleston personnel during reference (f) of APPENDIX A, data were obtained to estimate the value of T_1 based on local procedures. The result is based on experience to a degree and should be verified using a larger data base. The value for T_1 is composed of:

$$\frac{\text{vehicle cost + mileage + personnel cost}}{\text{units disposed}}$$

$$T_1 = \left[\left(\frac{\$1.32}{\text{Hour}} \times \frac{\text{1 stop/disposal}}{\text{5 stops/hour}}\right) + \left(\frac{\$.21}{\text{Mile}} \times \frac{\text{3 miles}}{\text{disposal}}\right) + \left(\frac{\$1.72 \text{ salary and fringe}}{\text{disposal action}}\right)\right] \div \frac{\text{50 units}}{\text{average disposal action}}$$

$$= \frac{\$.27 + \$.63 + \$1.72}{50} = \$0.05/\text{to dispose one unit}$$

7. The transportation cost to move material to a repair site, symbol T_2, is also peculiar to the proposed model and is not specified in any known directive. Through the courtesy of NAVMTO (Navy Material Transportation Office), a portion of the data was obtained to determine the value of T_2. Other data was obtained from the UICP SIG (Selective Item Generator) files.

From NAVMTO:

THEATER	DATES OF SHIPMENT	NR OF SHIPMENTS	COST TO SHIP	AVG UNITS/ SHIPMENT
Atlantic	3/10 - 7/20/78	814	$19,791.72	2
Pacific	4/21 - 6/29/78	212	3,266.44	2

From SIG Files:

DATE OF SIG	ICP	COG	MOS. OF DATA	CARCASSES RETURNED
9/76	ASO	2R	3	94,718
1/77	SPCC	2H,4A,4G,4N,6G,6U	1	7,108

The carcasses shipped to CONUS (Continental United States) repair sites from each theater was estimated as follows:

Atlantic: $\frac{2 \text{ units}}{\text{shipment}} \times \frac{814 \text{ shipments}}{4 \text{ months}} \times \frac{12 \text{ months}}{\text{year}} = 4{,}884$ units/year

Pacific: $\frac{2 \text{ units}}{\text{shipment}} \times \frac{212 \text{ shipments}}{2 \text{ months}} \times \frac{12 \text{ months}}{\text{year}} = 2{,}544$ units/year

Total: 7,428 units/year

The total carcasses shipped (CONUS and EX-CONUS) is estimated as follows:

$$\text{ASO:} \quad \frac{94{,}718 \text{ units}}{3 \text{ months}} \times \frac{12 \text{ months}}{\text{year}} = 378{,}872 \text{ units/year}$$

$$\text{SPCC:} \quad \frac{7{,}108 \text{ units}}{1 \text{ month}} \times \frac{12 \text{ months}}{\text{year}} = 85{,}296 \text{ units/year}$$

Total: 464,168 units/year

The estimated percentage of carcasses shipped from EX-CONUS is: 7,428/464,168 = .016 or 1.6%.

The value of T_2 is the CONUS cost (5¢/unit previously computed) plus the unit cost to ship to CONUS.

$$T_2 = .016 \left[\frac{\frac{\$3266.44}{212 \times 2} + \frac{\$19{,}791.72}{814 \times 2}}{2} \right] + .984(.05) = \$.21/\text{unit}$$

8. The disposal return rate, symbol p, for RFI material was estimated by the Defense Property Disposal Officer in Columbus, Ohio to be 5% for RFI material. No value was provided for disposal of NRFI material, but for purposes of this study, a value of 2% was assumed; the symbol used is j.

9. The repair administrative cost, symbol A_3, is estimated to be \$102 for SPCC and \$14.96 for ASO.

10. Shelf life values, symbol S, are available from UICP files. Items are assigned a shelf life according to a shelf life code assigned. Some examples:

SHELF LIFE CODE	SHELF LIFE (S)
A	.08
D	.33
J	1.25
L	1.75
P	2.50
X	5.00

11. The administrative cost of a disposal action, symbol β, was estimated for model test purposes using data from various sources.

COST FACTOR	SOURCE	COST/DISPOSAL ACTION
Disposal Directive Review Keypunching	SPCC SPCC	$ .24
Off-line AUTODIN Labor (ICP)	ALRAND 237	1.23
Off-line Labor (Stock Point) Computer Time (Stock Point) Computer Operator (Stock Point)	NSC Charleston NSC Charleston NSC Charleston	.57
Warehouse Labor (Stock Point)	NSC Charleston	16.80
TOTAL		$18.84

NOTE: Model testing showed β to have minimal impact on the solution variables t_1 and t_2 and was deleted from the final model formulation. See TABLE I, page 31.

III. MODEL EVALUATION

The proposed model, equations (5) and (7), was evaluated by sensitivity and empirical analysis. Sensitivity analysis uses hypothetical model parameters to evaluate the impact on the solution variables, t_1 and t_2. This is a theoretical test to assure the models are feasible and function properly. The empirical analysis uses actual ICP file data as input to the current model and the proposed model. Solutions for the current and proposed model are compared to measure the impact of the proposed model on the economic retention requirement. This is the practical test.

A. SENSITIVITY ANALYSIS. An initial task for the sensitivity analysis is the establishment of benchmark values for the models solution variables t_1 and t_2. Model parameters are chosen and the resultant values of t_1 and t_2 are computed for future reference. The sensitivity of the expected cost to hold material and the expected cost to reprocure disposed material are illustrated in FIGURES 3 and 4. The point of intersection of the curves indicate the optimal number of years of material to hold in retention. For t_1 in FIGURE 3, 8.25 years of forecasted demand is the economic retention quantity and for t_2 in FIGURE 4, 7.77 years of forecasted demand is the economic retention quantity. Use of these values is shown on FIGURE B-1, APPENDIX B.

TABLE I indicates the variable β, as noted earlier, appears to have minimal impact on the solution variable. For this reason, β was dropped from the models.

The sensitivity of t_1 and t_2 to variations in the model variables are shown in the tables and graphs of APPENDIX B. The steeper the slope of the curve the greater the sensitivity of the solution variable to that parameter. Many of the parameters

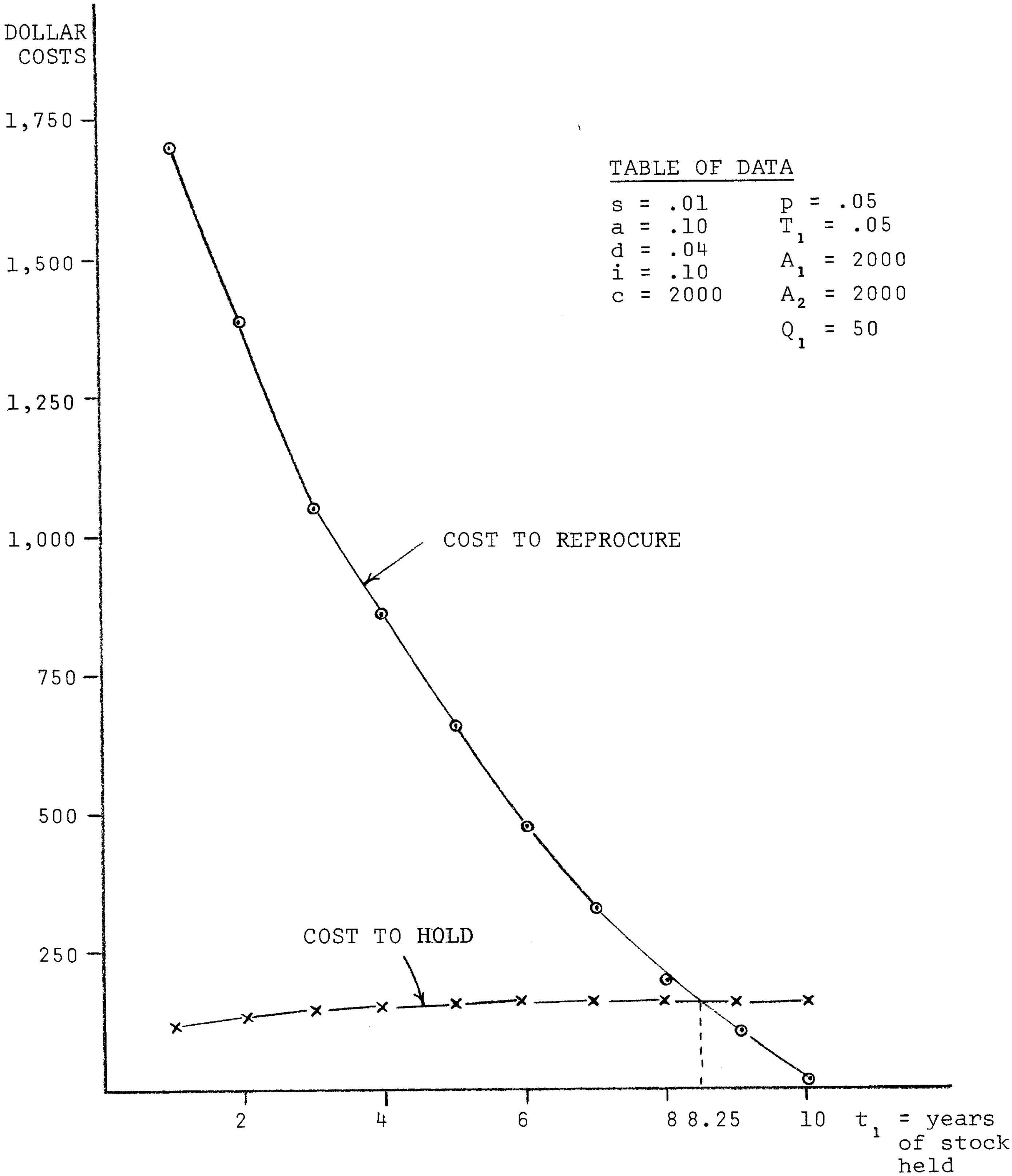
DOLLAR COSTS
1,750
1,500
1,250
1,000
750
500
250
TABLE OF DATA
s = .01
a = .10
d = .04
i = .10
c = 2000
p = .05
T1 = .05
A1 = 2000
A2 = 2000
Q1 = 50
COST TO REPROCURE
COST TO HOLD
2
4
6
8 8.25
10
t1 = years of stock held

FIGURE 3

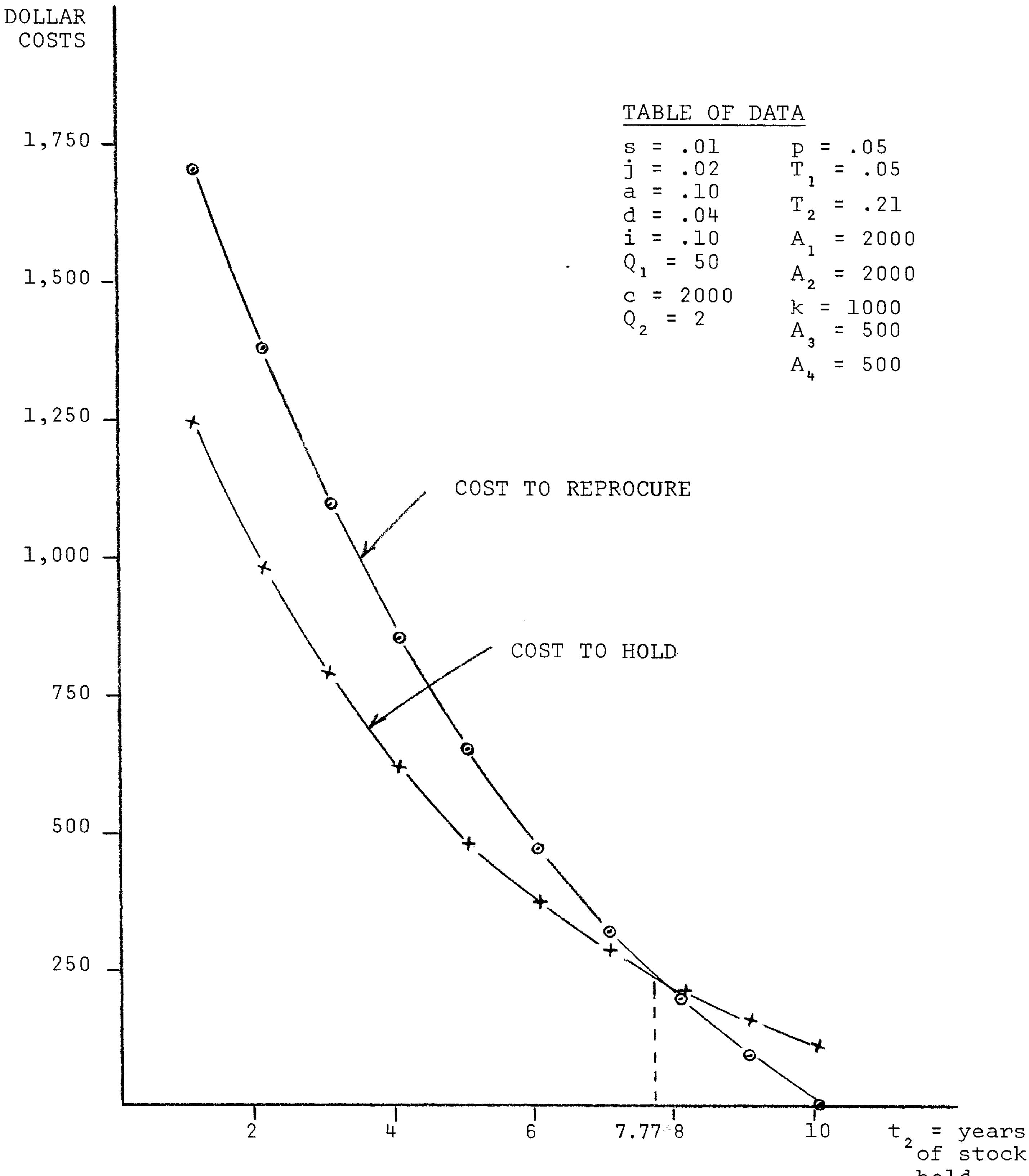
DOLLAR COSTS
1,750
1,500
1,250
1,000
750
500
250
TABLE OF DATA
s = .01
j = .02
a = .10
d = .04
i = .10
Q1 = 50
c = 2000
Q2 = 2
p = .05
T1 = .05
T2 = .21
A1 = 2000
A2 = 2000
k = 1000
A3 = 500
A4 = 500
COST TO REPROCURE
COST TO HOLD
2
4
6
7.77
8
10
t2 = years of stock held

FIGURE 4

TABLE I

SENSITIVITY OF t_1 AND t_2 WITH β VARIED

	s	j	a	d	i	p	β	D_g	X_1	X_2	A_1	A_2	Q_1	c	k	i'	A_3	A_4	Q_2	t_1	$\%\Delta t_1$	t_2	$\%\Delta t_2$
Benchmark	.01	.02	.1	.04	.1	.05	0	400	.05	.21	2000	2000	50	2000	1000	.1	500	500	2	8.25	0	7.77	0
	.01	.02	.1	.04	.1	.05	19	400	.05	.21	2000	2000	50	2000	1000	.1	500	500	2	8.25	0	7.77	0
	.01	.02	.1	.04	.1	.05	5010	400	.05	.21	2000	2000	50	2000	1000	.1	500	500	2	8.26	0	7.80	0

have small absolute values, thus great care must be exercised in assigning values because small errors can produce large errors in the values of t_1 and t_2.

B. EMPIRICAL ANALYSIS. The proposed model was given a simulated operational test using UICP data from the September 1978 stratification at ASO and SPCC. The data was reviewed for accuracy by the ICP personnel prior to use. To evaluate the proposed rules against the existing, the model was constrained using equations (15) through (21). To identify/differentiate various model outputs, the current model constrained value was labelled ERR, and the proposed model constrained value was labelled ERR2. Specific statistics for evaluation are:

1. Value of proposed model with new constraints (ERR2).
2. Value of proposed model with current constraints (ERR1).
3. Value of current model with current constraints (ERR).
4. Difference between current constrained economic retention quantity and proposed model with current constrained retention quantity (ERR-ERR1).
5. Difference between current constrained economic retention quantity and the proposed constrained economic retention quantity (ERR-ERR2).

Frequency distributions of these values were developed for major inventory segments; 1R and 2R at ASO, and 1H, 2H, 4G, and 4N at SPCC. The statistics were developed assuming a zero disposal rate and then a disposal rate of 5% of material standard price for RFI and 2% for NRFI. The items in categories MARK 0, MARK I, MARK II, and Provisioned items were not included in the comparison due to lack of consideration of the model solution variables (t_1 and t_2) in the current model. TABLE II shows a count of total items by MARK that were used in this analysis.

TABLE II

ITEM COUNTS BY ICP, COG, MARK's 0, I, II, AND PROVISIONED ITEMS

ITEM CATEGORY	SPCC	ASO
Total items in Universe	334,363	223,245
MARK 0 Items	252,865	154,348
Non-MARK 0 Items	81,498	68,897
1H	50,112	-
2H	8,668	-
4G	6,698	-
4N	6,672	-
1R	-	48,470
2R	-	14,798
All Others	9,348	5,629
Provisioned Items	13,867	6,812
Non-MARK 0; Not Provisioned Items	67,631	62,085
1H	47,491	-
2H	5,158	-
4G	3,402	-
4N	4,366	-
1R	-	45,922
2R	-	12,100
All Others	7,214	4,063
MARK I and II Consumable Items	22,644	11,577
Non-MARK 0; Not Provisioned; Non-MARK I and II Consumables	44,987	50,508
1H	25,559	-
2H	5,097	-
4G	3,381	-
4N	4,356	-
1R	-	35,373
2R	-	12,100
All Others	6,594	3,035

The details of the analysis are shown in APPENDIX C.

TABLES C-1 through C-3 (APPENDIX C) computed the various retention quantities (new model/new constraints = ERR2; new model/

current constraints = ERR1; and current model with current constraints = ERR). A value of p = 0 was used which is the understood NAVSUP policy. This is based on the lack of financial return to the Navy since Defense Property Disposal Office proceeds go to the U. S. Treasury. Economic arguments can extend the realm of consideration to justify using a value for p. TABLES C-1 and C-2 contain frequency distributions for the priced out values of the economic retention requirement using current and alternative models for major SPCC cogs. TABLE C-3 shows the differences among the three models. A higher economic retention requirement is indicated under the current constraints and the proposed model for 1H, 6G, and 6M cogs with a lower economic retention requirement indicated for the other cogs. ERR1 in total indicates a requirement $16,471,021 lower than ERR in TABLE C-3. TABLE C-3 indicates ERR2 computes a much higher retention requirement than ERR, or $2,610,730,124 for SPCC cogs.

TABLES C-4, C-5, and C-6 provide the same analysis on ASO material that TABLES C-1, C-2, and C-3 did on SPCC cogs. The analysis shows that ERR1 gives a higher economic retention requirement for 1R and 5R material, but a lower requirement for the balance of the inventory segments. Overall ERR1 gave a requirement $68,725 higher than ERR. ERR2 gave a retention requirement $5,734,396,305 higher than ERR.

TABLES C-7 through C-12 provide similar analysis except that values of p = .05 and j = .02 were used vice current values of zero. The results for comparing the proposed model (ERR1) with the current (ERR) using the same constraints indicates that ERR1 requirements compute $24,320,778 less for SPCC. TABLE C-9 also shows that ERR2 (proposed model/new constraints) computes an economic retention requirement of $1,772,778,470 more than ERR. TABLE C-12 shows ERR computes a requirement $8,836,429 higher than ERR1, but $2,965,996,470 less than ERR2 for ASO inventories using values of p = .05 and j = .02 vice zero.

The proposed model using current constraints computes lower economic retention requirements than the current model. Using the alternative constraints and the proposed model, the retention requirement computed is much higher than current retention requirement. The subtotals of TABLES C-3, C-6, C-9, and C-12 indicate the relative results, as follows:

a. ERR-ERR1.

ICP	ASSUMPTION	Δ INVESTMENT
SPCC	p=0; j=0	$16,471,021
ASO	p=0; j=0	68,725
TOTAL		$16,539,746
SPCC	p=.05; j=.02	$24,320,778
ASO	p=.05; j=.02	8,836,429
TOTAL		$33,157,207

b. ERR-ERR2.

ICP	ASSUMPTION	Δ INVESTMENT
SPCC	p=0; j=0	$-2,610,730,124
ASO	p=0; j=0	-5,734,396,305
TOTAL		$-8,345,126,429
SPCC	p=.05; j=.02	$-1,772,778,470
ASO	p=.05; j=.02	-2,965,996,470
TOTAL		$-4,738,774,940

IV. SUMMARY

The dynamics of fleet operations and changes in supply policy tend to cause change in demand forecasts and fluctuation in material excesses. The current Navy economic retention model was designed to compute the optimal amount of excess consumable item assets to hold in retention based on economic criteria. This study develops an improved economic retention model applicable to repairable and consumable items. The model formulation considers both the costs to hold assets and to reprocure assets disposed prematurely.

The proposed model was evaluated with constraints used in the current model and with alternative constraints introduced into the proposed model to consider repairable items and other economic factors. Current constraints, such as shelf life, were retained, but transportation costs were introduced, as examples.

The proposed model was evaluated initially in a theoretical mode to assure the feasibility of the proposed mathematical technique. The model design eliminates the approximations used in the current model and provides a more precise solution. The binary search routine was shown to provide precision using actual data. The theoretical examination also examined the sensitivity of the various parameters in the model. The model solution variables t_1 and t_2 (optimum years of demand to retain) were found to be sensitive to changes in disposal transportation cost, repair price, repair administrative cost, and repair set-up cost as these values became large. Several new model variables were tested with values based upon experience, which should be determined more accurately. Other parameter values, though small in absolute value, had profound impact on the values of t_1 and t_2.

A practical test of the model consisted of an empirical examination of the differences between the current and proposed models using ICP stratification data. Under current constraints the proposed model retains less material (lower economic retention requirement) than the current model. The proposed model using the alternative economic constraints computes a higher economic retention requirement than the current model.

The proposed economic retention model represents an improvement over the current model. Approximations are eliminated and repairable items considered along with consumable items. The mathematics exist to implement the model as a portion of the stratification process. Additional study will be required to establish optimum values of parameter values and procedures must then be established to maintain the proper values.

APPENDIX A: REFERENCES

(a) ALRAND Report 45 - "Inventory Control Manual - The Uniform Automated Data Processing System" of 12 Apr 1965.

(b) FMSO ltr F9222-D36/JWS/115 5250 of 7 Mar 1978.

(c) First Course in Numerical Methods, Walter Jennings, The MacMillan Company, 1964.

(d) DODINST 4140.39 of 17 Jul 1970.

(e) Telcon between Mr. J. Harding (FMSO 932) and Mr. C. Goss (NSC Charleston, 43) on 27 Jul 1978.

(f) Telcon between Mr. J. Harding (FMSO 932) and Mr. R. Farley (NSC Charleston, 407) on 27 Jul 1978.

APPENDIX B: RESULTS OF SENSITIVITY ANALYSIS

1.	FIGURE B-1	s varied
2.	FIGURE B-2	j varied
3.	FIGURE B-3	a varied
4.	FIGURE B-4	d varied
5.	FIGURE B-5	i varied
6.	FIGURE B-6	p varied
7.	FIGURE B-7	T_1 varied
8.	FIGURE B-8	T_2 varied
9.	FIGURE B-9	A_1 varied
10.	FIGURE B-10	A_2 varied
11.	FIGURE B-11	Q_1 varied
12.	FIGURE B-12	c varied
13.	FIGURE B-13	k varied
14.	FIGURE B-14	A_3 varied
15.	FIGURE B-15	A_4 varied
16.	FIGURE B-16	Q_2 varied

SENSITIVITY OF t_1 AND t_2 WITH s VARIED

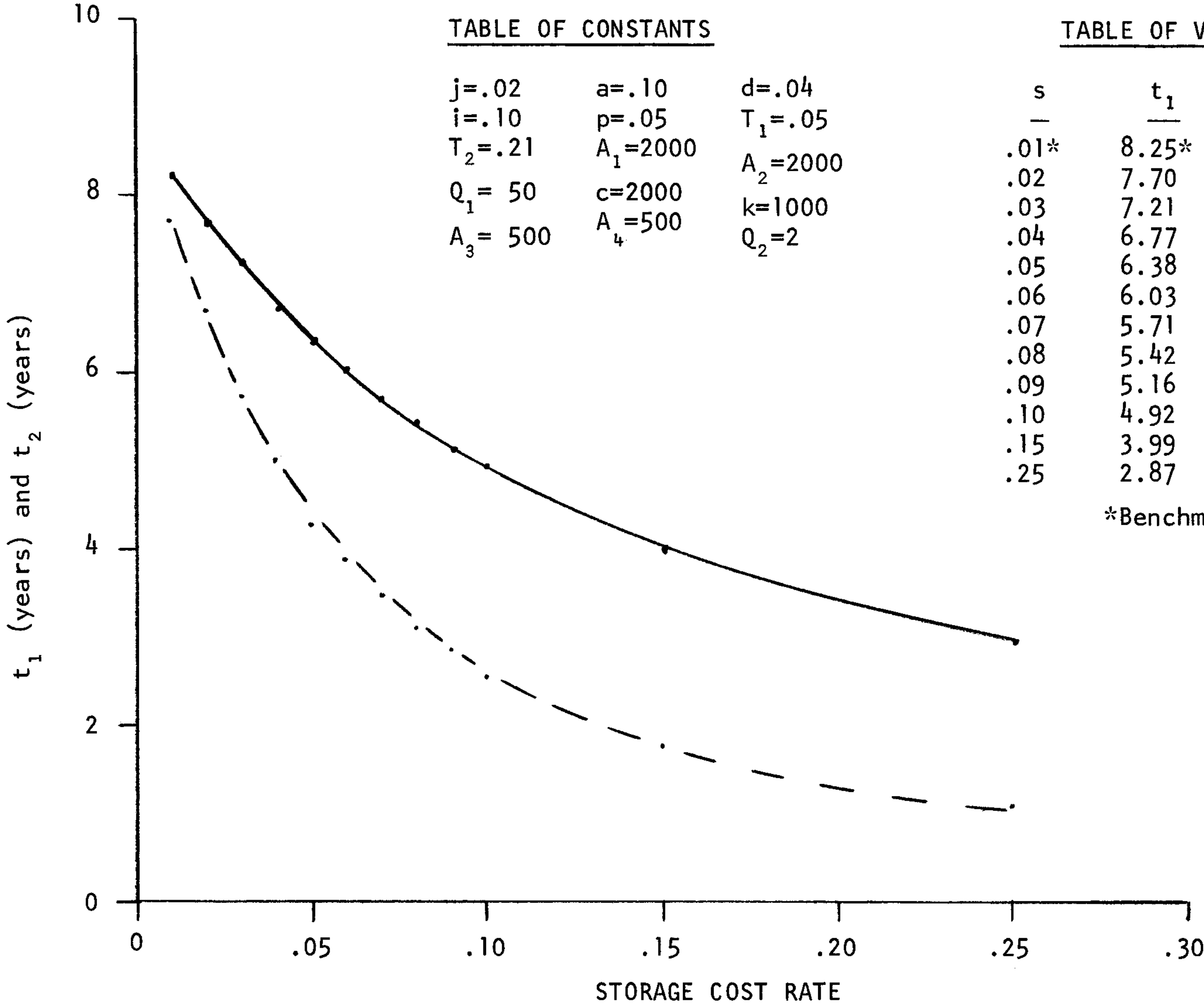

TABLE OF CONSTANTS

$j=.02$	$a=.10$	$d=.04$
$i=.10$	$p=.05$	$T_1=.05$
$T_2=.21$	$A_1=2000$	$A_2=2000$
$Q_1=50$	$c=2000$	$k=1000$
$A_3=500$	$A_4=500$	$Q_2=2$

TABLE OF VARIABLES

s	t_1	t_2
.01*	8.25*	7.77*
.02	7.70	6.67
.03	7.21	5.75
.04	6.77	5.00
.05	6.38	4.39
.06	6.03	3.88
.07	5.71	3.47
.08	5.42	3.12
.09	5.16	2.83
.10	4.92	2.59
.15	3.99	1.79
.25	2.87	1.09

*Benchmark

FIGURE B-1

SENSITIVITY OF t_2 WITH j VARIED

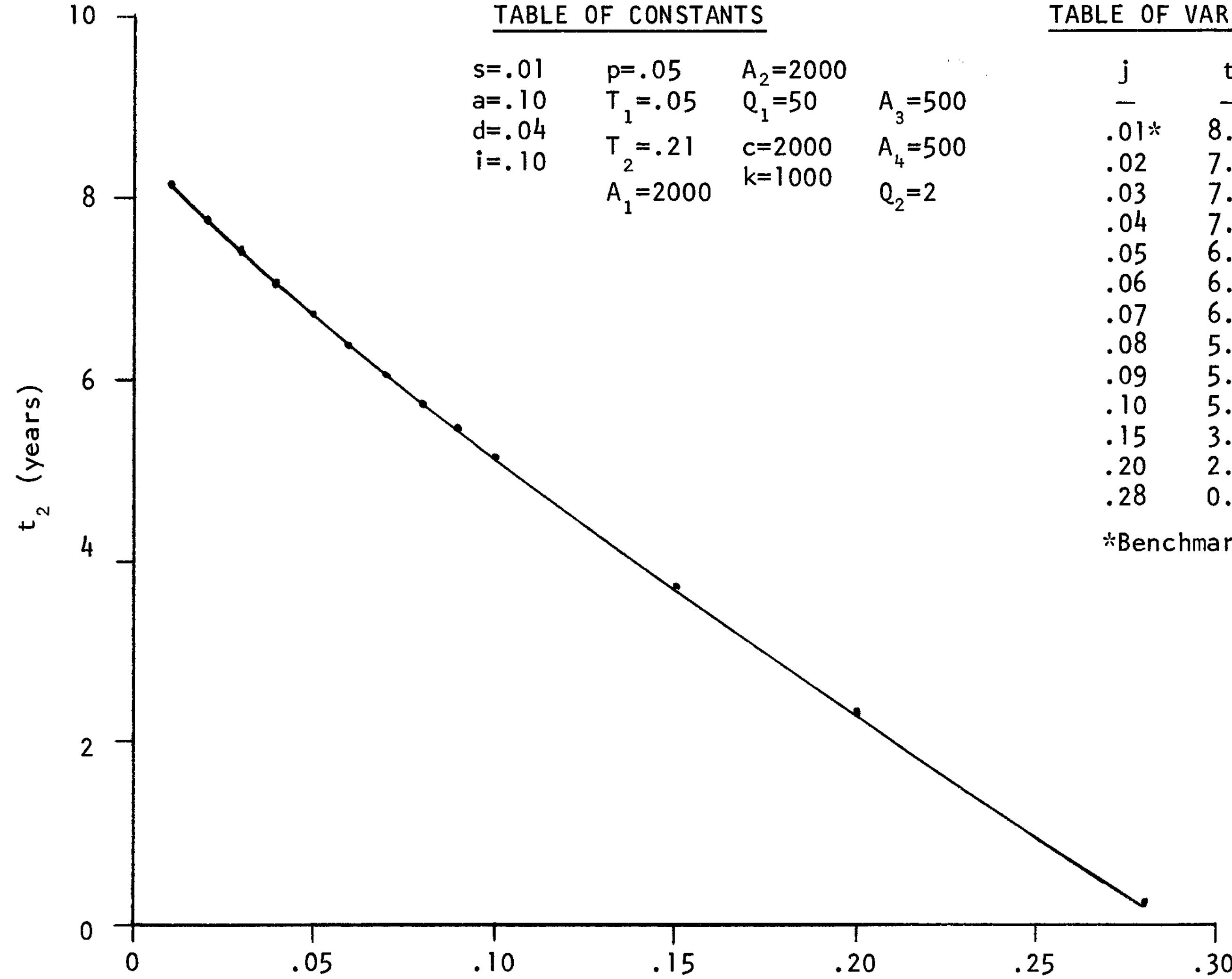

TABLE OF CONSTANTS

s=.01	p=.05	A_2=2000	
a=.10	T_1=.05	Q_1=50	A_3=500
d=.04	T_2=.21	c=2000	A_4=500
i=.10	A_1=2000	k=1000	Q_2=2

TABLE OF VARIABLES

j	t_2
.01*	8.15*
.02	7.77
.03	7.41
.04	7.05
.05	6.71
.06	6.38
.07	6.06
.08	5.75
.09	5.44
.10	5.14
.15	3.72
.20	2.39
.28	0.27

*Benchmark

DISPOSAL RETURN RATE FOR NRFI MATERIAL

FIGURE B-2

SENSITIVITY OF t_1 AND t_2 WITH a VARIED

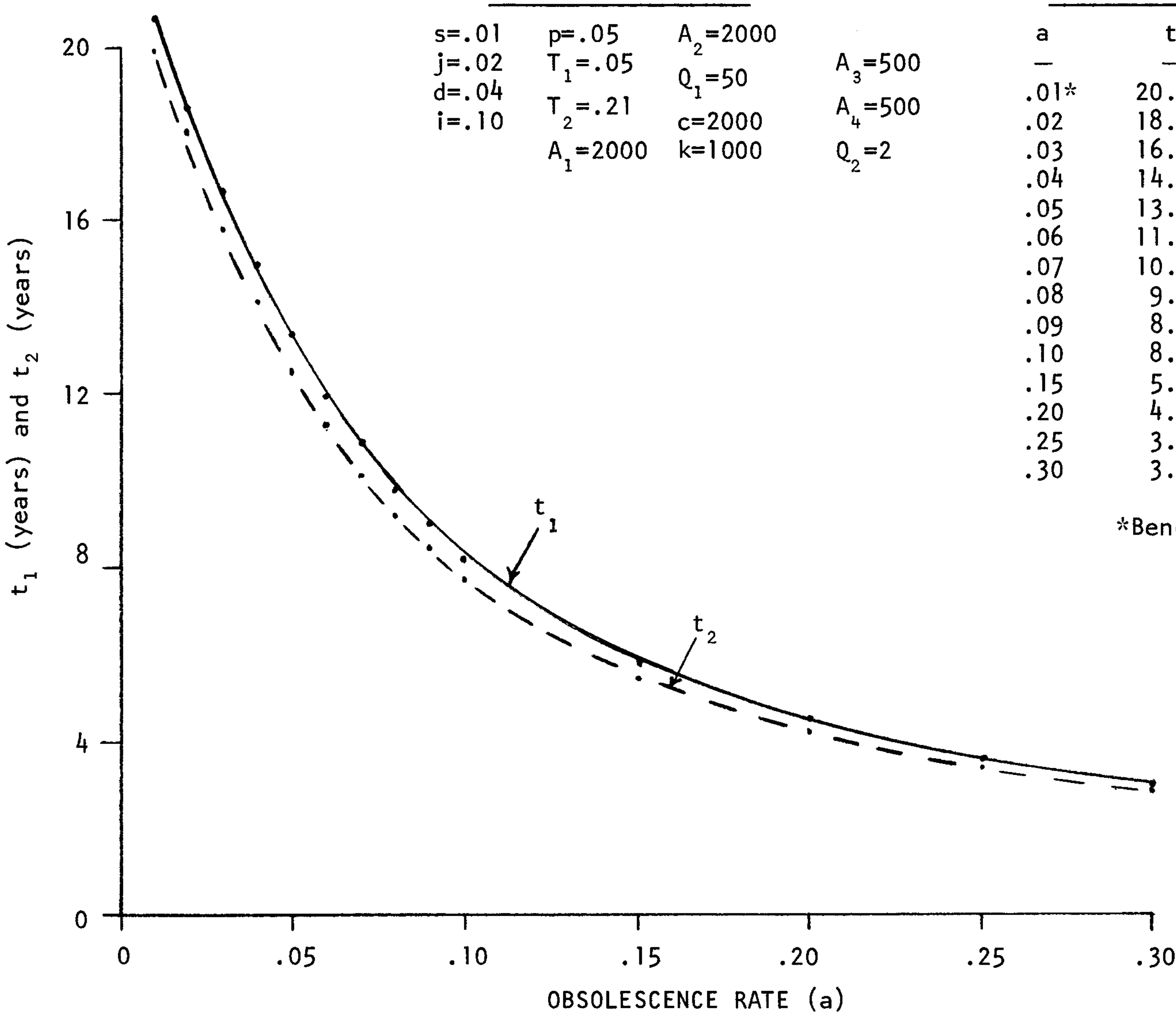

TABLE OF CONSTANTS

s=.01	p=.05	A_2=2000	
j=.02	T_1=.05	Q_1=50	A_3=500
d=.04	T_2=.21	c=2000	A_4=500
i=.10	A_1=2000	k=1000	Q_2=2

TABLE OF VARIABLES

a	t_1	t_2
.01*	20.68*	19.96*
.02	18.68	17.86
.03	16.73	15.88
.04	14.94	14.10
.05	13.35	12.56
.06	11.98	11.25
.07	10.81	10.15
.08	9.82	9.23
.09	8.97	8.44
.10	8.25	7.77
.15	5.83	5.52
.20	4.48	4.27
.25	3.64	3.48
.30	3.06	2.93

*Benchmark

FIGURE B-3

SENSITIVITY OF t_1 AND t_2 WITH d VARIED

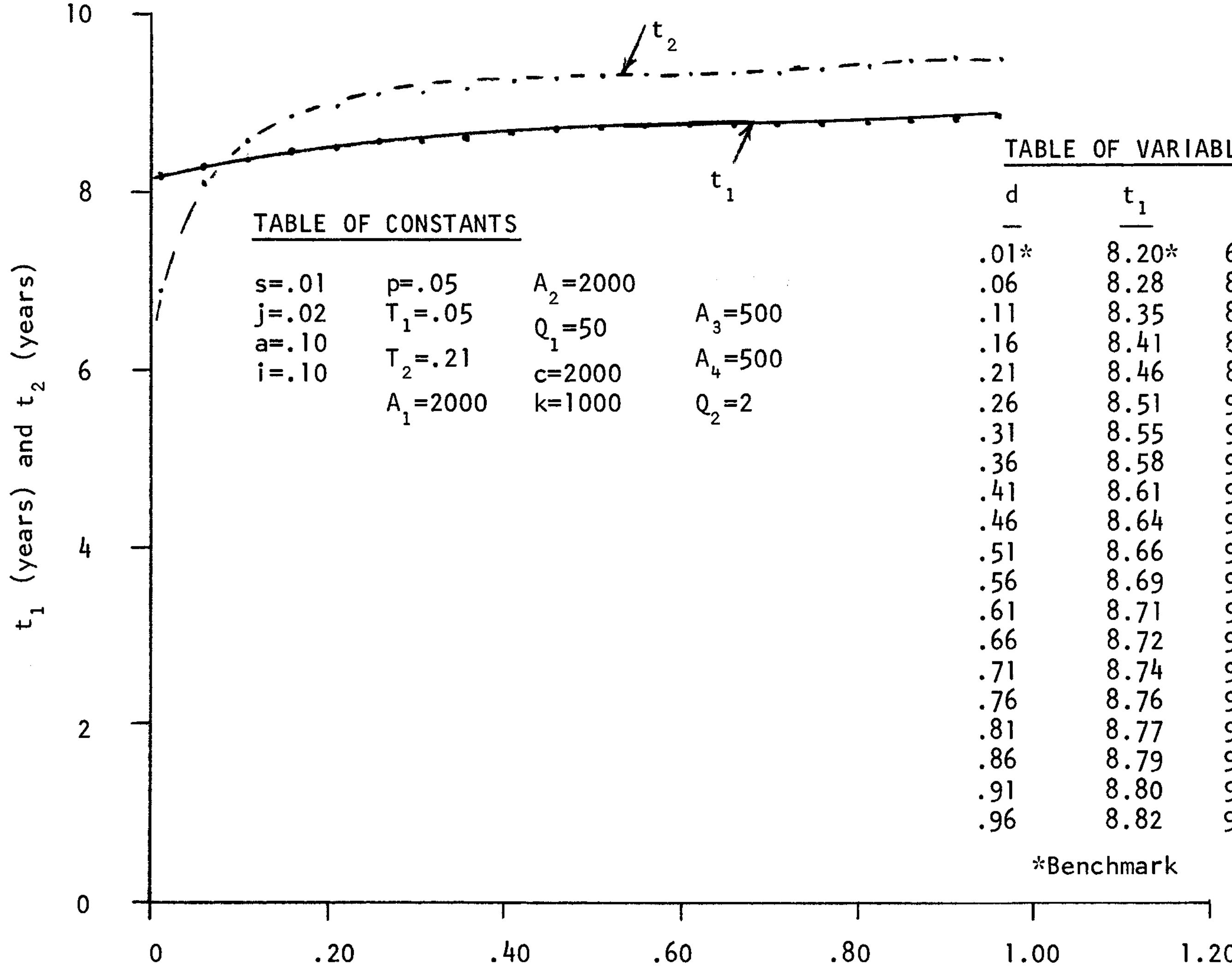

TABLE OF CONSTANTS

s=.01	p=.05	A_2=2000	
j=.02	T_1=.05	Q_1=50	A_3=500
a=.10	T_2=.21	c=2000	A_4=500
i=.10	A_1=2000	k=1000	Q_2=2

TABLE OF VARIABLES

d	t_1	t_2
.01*	8.20*	6.92*
.06	8.28	8.11
.11	8.35	8.59
.16	8.41	8.83
.21	8.46	8.98
.26	8.51	9.07
.31	8.55	9.13
.36	8.58	9.18
.41	8.61	9.22
.46	8.64	9.25
.51	8.66	9.28
.56	8.69	9.31
.61	8.71	9.33
.66	8.72	9.35
.71	8.74	9.37
.76	8.76	9.39
.81	8.77	9.40
.86	8.79	9.42
.91	8.80	9.44
.96	8.82	9.46

*Benchmark

PROBABILITY OF INVENTORY LOSS IN STORAGE

FIGURE B-4

SENSITIVITY OF t_1 AND t_2 WITH i VARIED

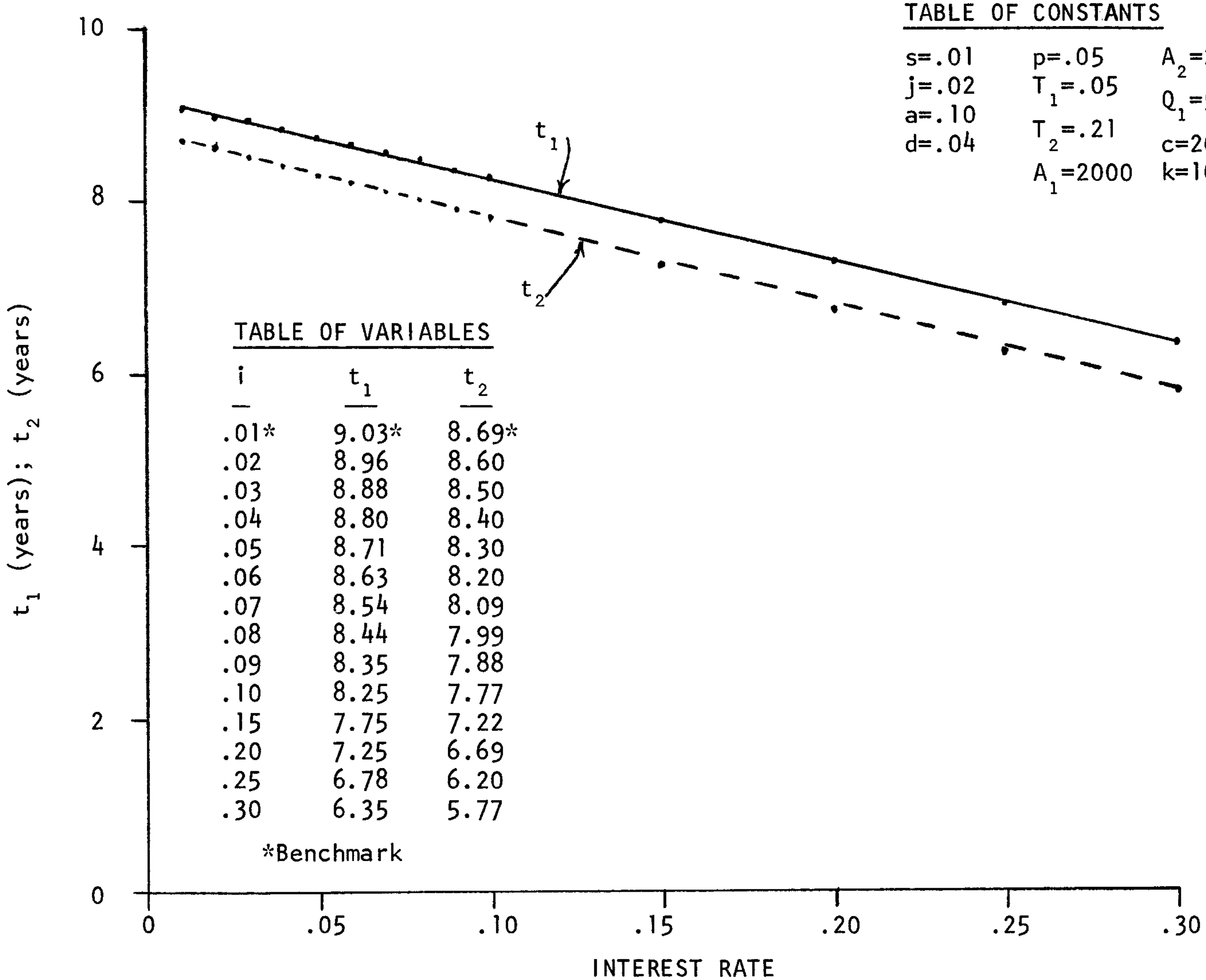

TABLE OF CONSTANTS

s=.01	p=.05	A_2=2000	A_3=500
j=.02	T_1=.05	Q_1=50	A_4=500
a=.10	T_2=.21	c=2000	Q_2=2
d=.04	A_1=2000	k=1000	

TABLE OF VARIABLES

i	t_1	t_2
.01*	9.03*	8.69*
.02	8.96	8.60
.03	8.88	8.50
.04	8.80	8.40
.05	8.71	8.30
.06	8.63	8.20
.07	8.54	8.09
.08	8.44	7.99
.09	8.35	7.88
.10	8.25	7.77
.15	7.75	7.22
.20	7.25	6.69
.25	6.78	6.20
.30	6.35	5.77

*Benchmark

FIGURE B-5

SENSITIVITY OF t_1 WITH p VARIED

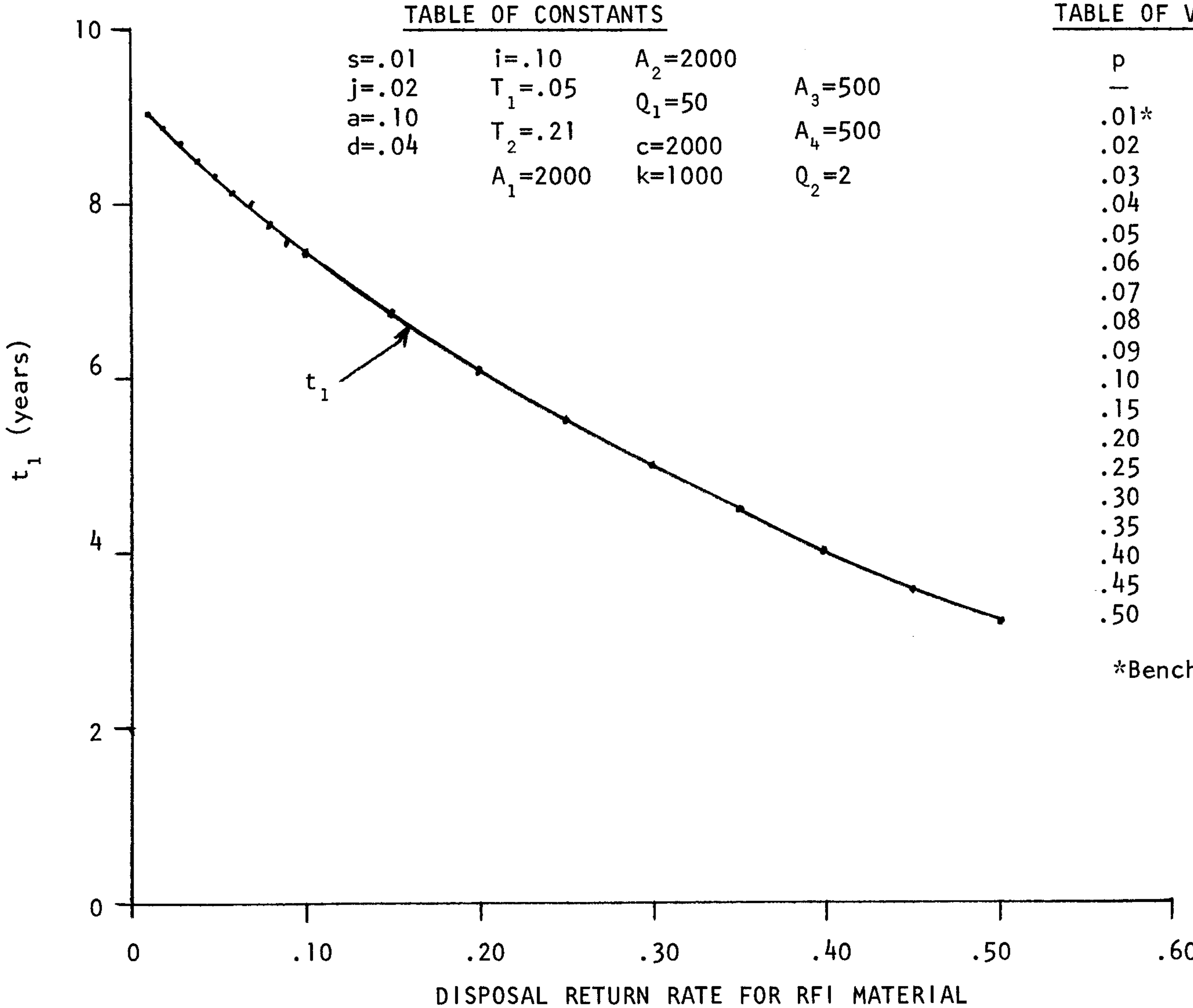

TABLE OF CONSTANTS

s=.01	i=.10	A_2=2000	
j=.02	T_1=.05	Q_1=50	A_3=500
a=.10	T_2=.21	c=2000	A_4=500
d=.04	A_1=2000	k=1000	Q_2=2

TABLE OF VARIABLES

p	t_1
.01*	9.02*
.02	8.82
.03	8.62
.04	8.43
.05	8.25
.06	8.07
.07	7.90
.08	7.74
.09	7.58
.10	7.42
.15	6.70
.20	6.05
.25	5.47
.30	4.95
.35	4.46
.40	4.01
.45	3.59
.50	3.19

*Benchmark

FIGURE B-6

SENSITIVITY OF t_1 AND t_2 WITH T_1 VARIED

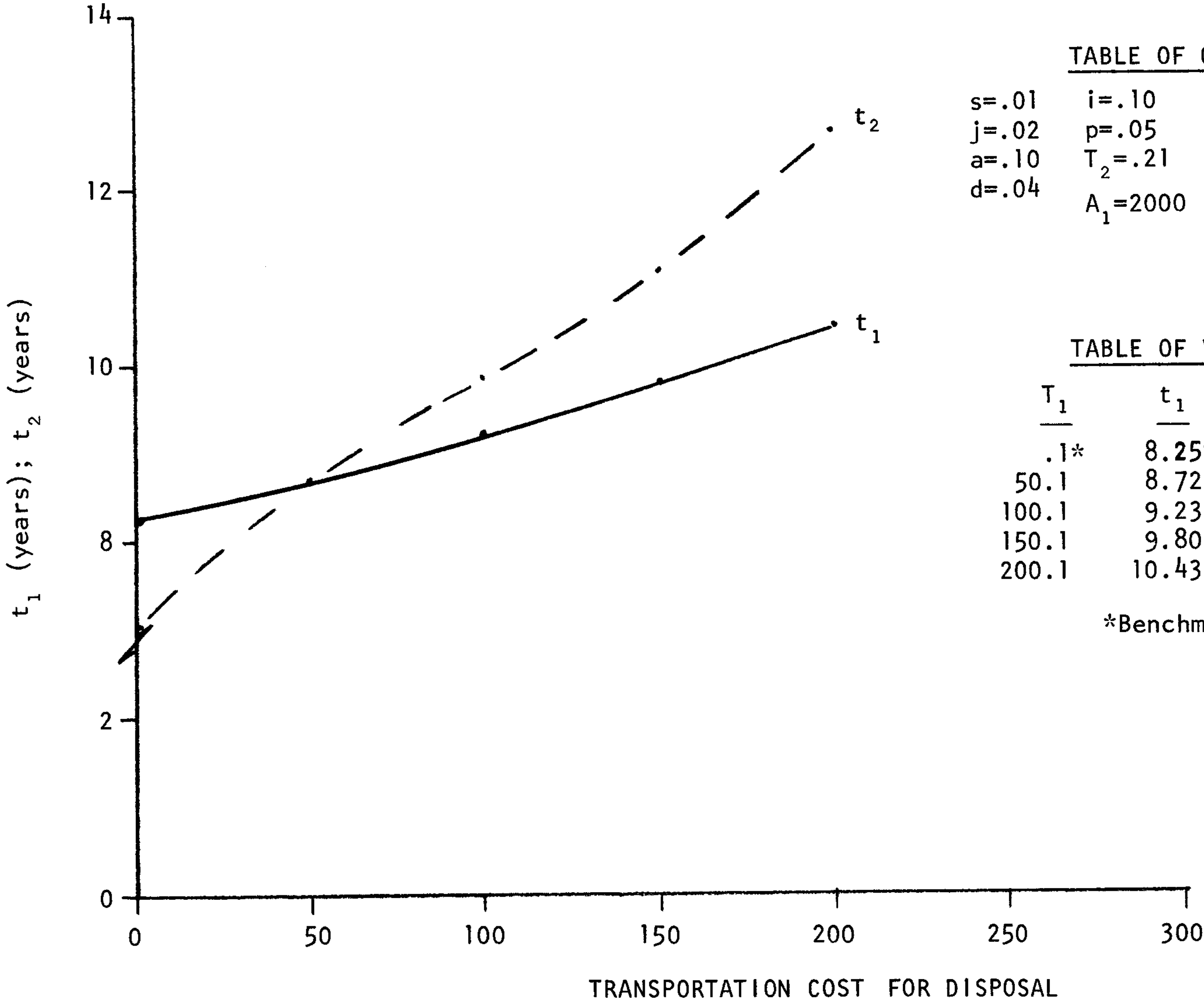

TABLE OF CONSTANTS

$s=.01$	$i=.10$	$A_2=2000$	$A_3=500$
$j=.02$	$p=.05$	$Q_1=50$	$A_4=500$
$a=.10$	$T_2=.21$	$c=2000$	$Q_2=2$
$d=.04$	$A_1=2000$	$k=1000$	

TABLE OF VARIABLES

T_1	t_1	t_2
.1*	8.25*	7.77*
50.1	8.72	8.74
100.1	9.23	9.84
150.1	9.80	11.12
200.1	10.43	12.68

*Benchmark

FIGURE B-7

SENSITIVITY OF t_2 WITH T_2 VARIED

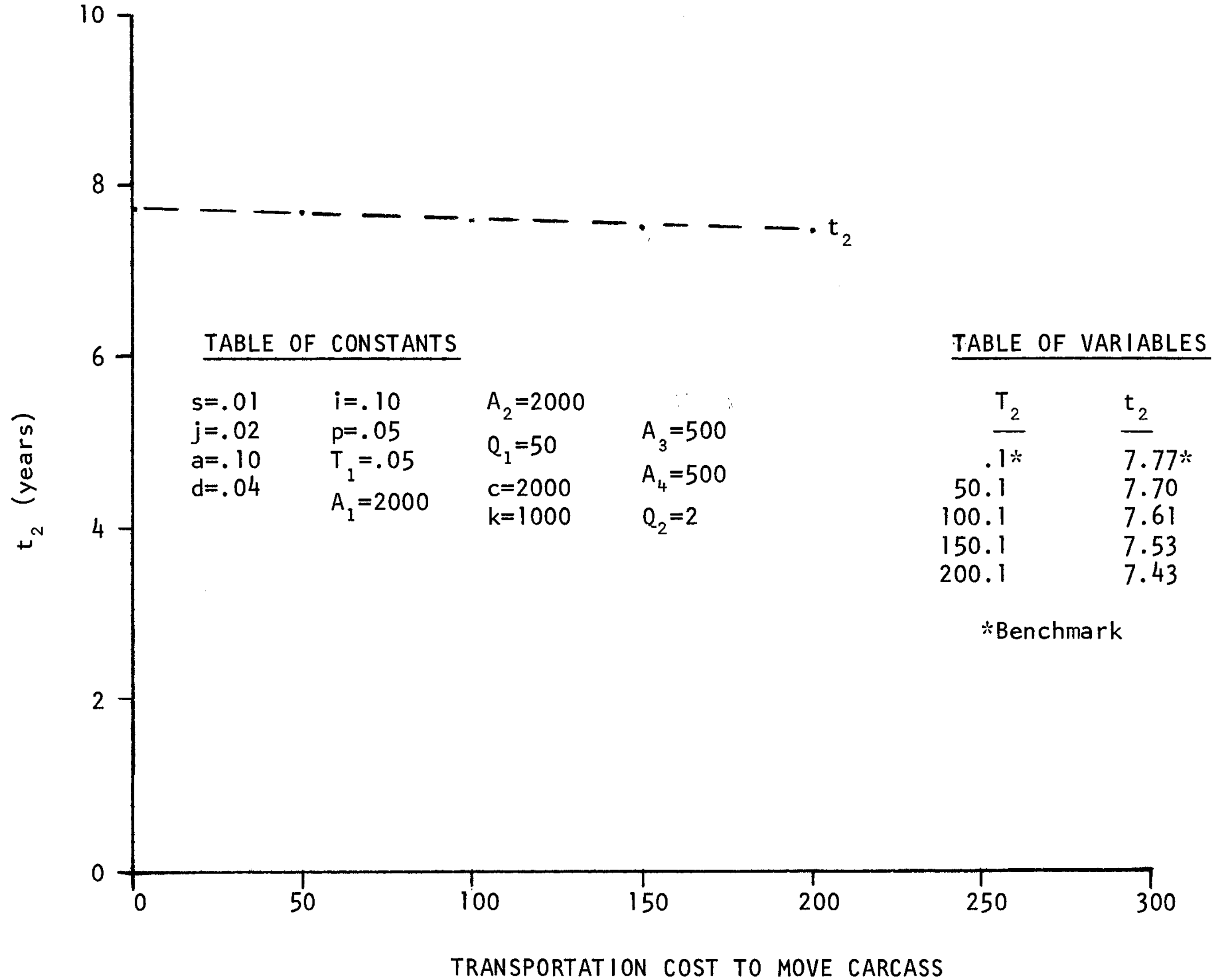

FIGURE B-8

SENSITIVITY OF t_1 AND t_2 WITH A_1 VARIED

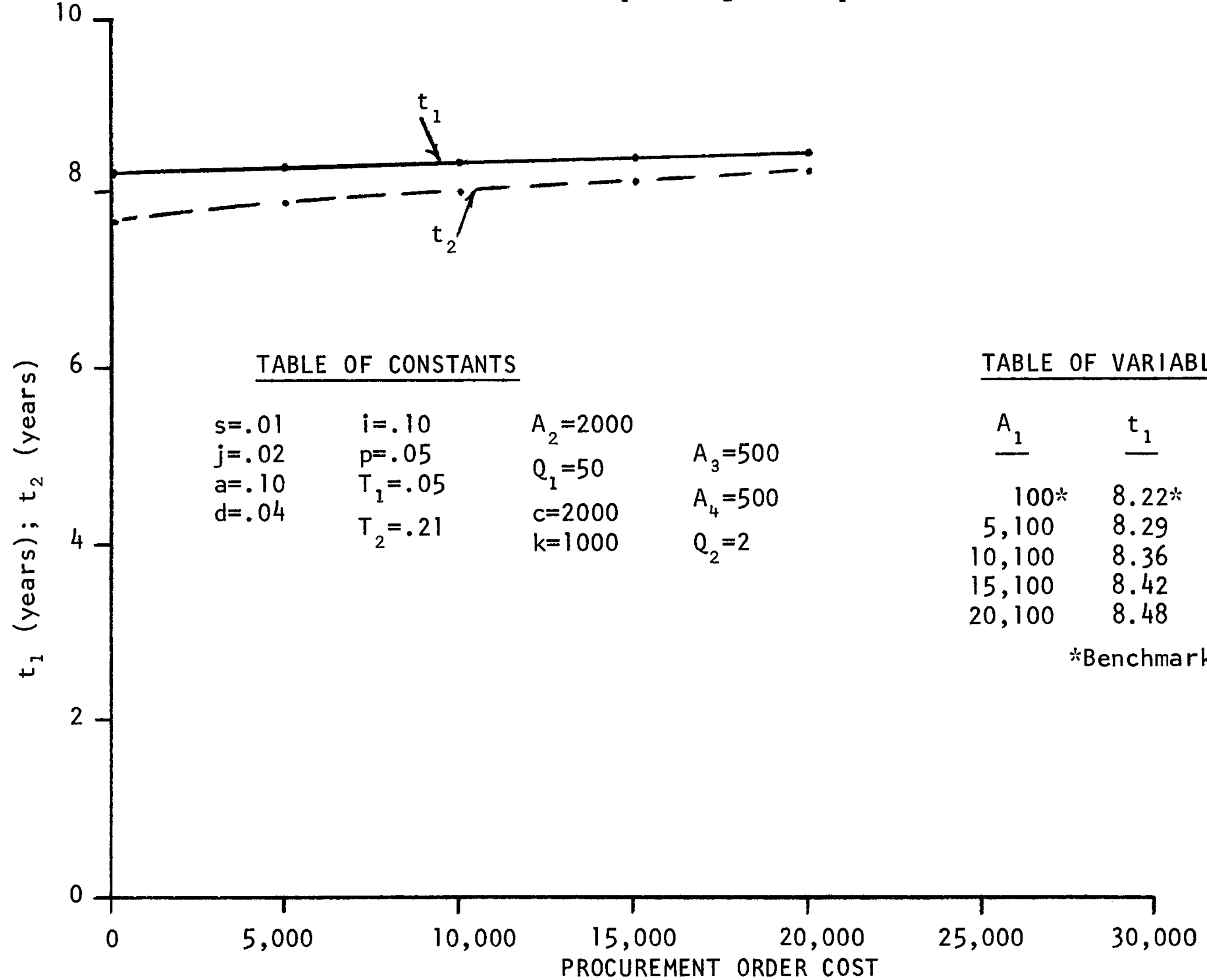

TABLE OF CONSTANTS

s=.01	i=.10	A_2=2000	A_3=500
j=.02	p=.05	Q_1=50	A_4=500
a=.10	T_1=.05	c=2000	Q_2=2
d=.04	T_2=.21	k=1000	

TABLE OF VARIABLES

A_1	t_1	t_2
100*	8.22*	7.69*
5,100	8.29	7.88
10,100	8.36	8.05
15,100	8.42	8.18
20,100	8.48	8.30

*Benchmark

FIGURE B-9

SENSITIVITY OF t_1 AND t_2 WITH A_2 VARIED

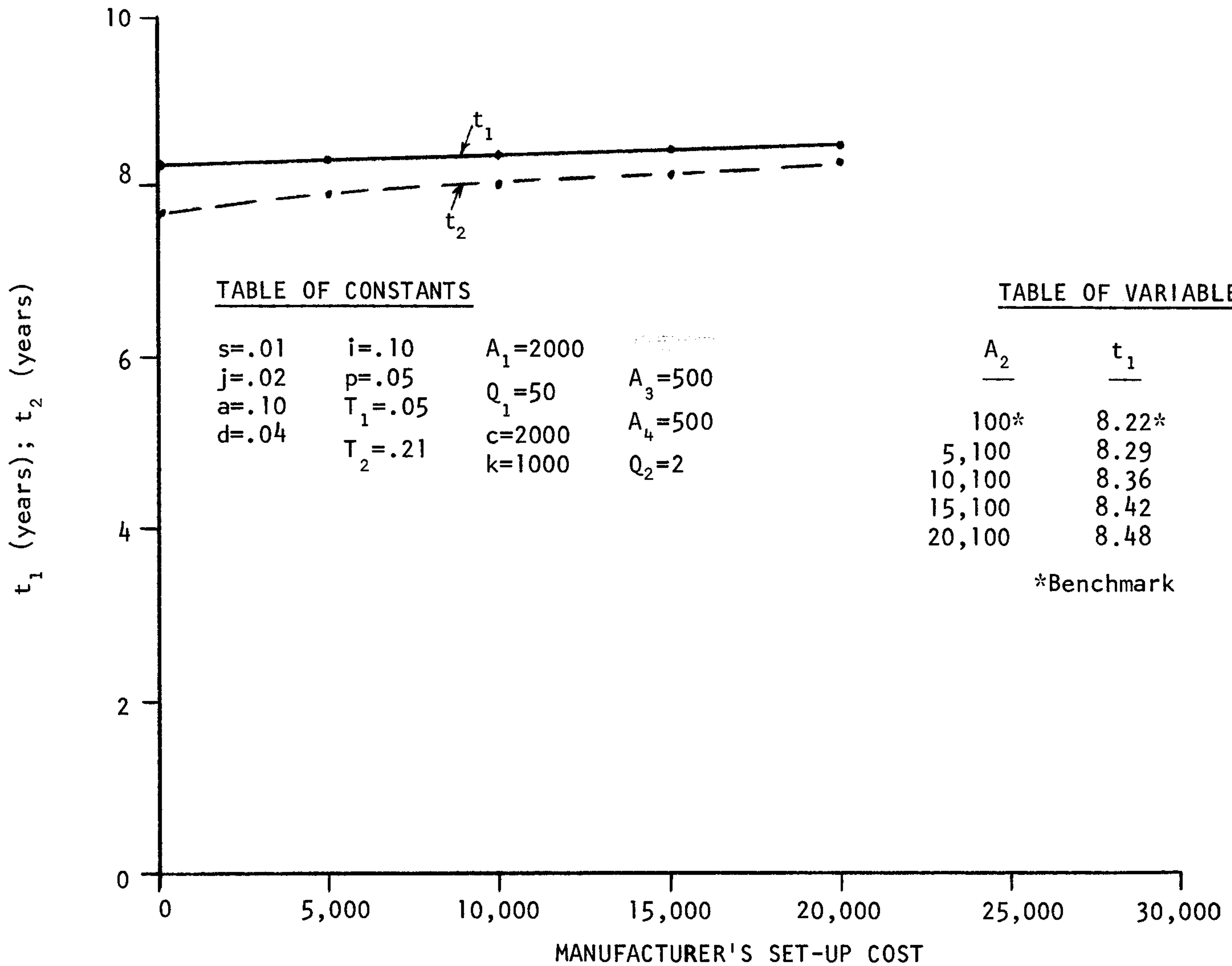

TABLE OF CONSTANTS

s=.01	i=.10	A_1=2000	
j=.02	p=.05	Q_1=50	A_3=500
a=.10	T_1=.05	c=2000	A_4=500
d=.04	T_2=.21	k=1000	Q_2=2

TABLE OF VARIABLES

A_2	t_1	t_2
100*	8.22*	7.69*
5,100	8.29	7.88
10,100	8.36	8.05
15,100	8.42	8.18
20,100	8.48	8.30

*Benchmark

FIGURE B-10

SENSITIVITY OF t_1 AND t_2 WITH Q_1 VARIED

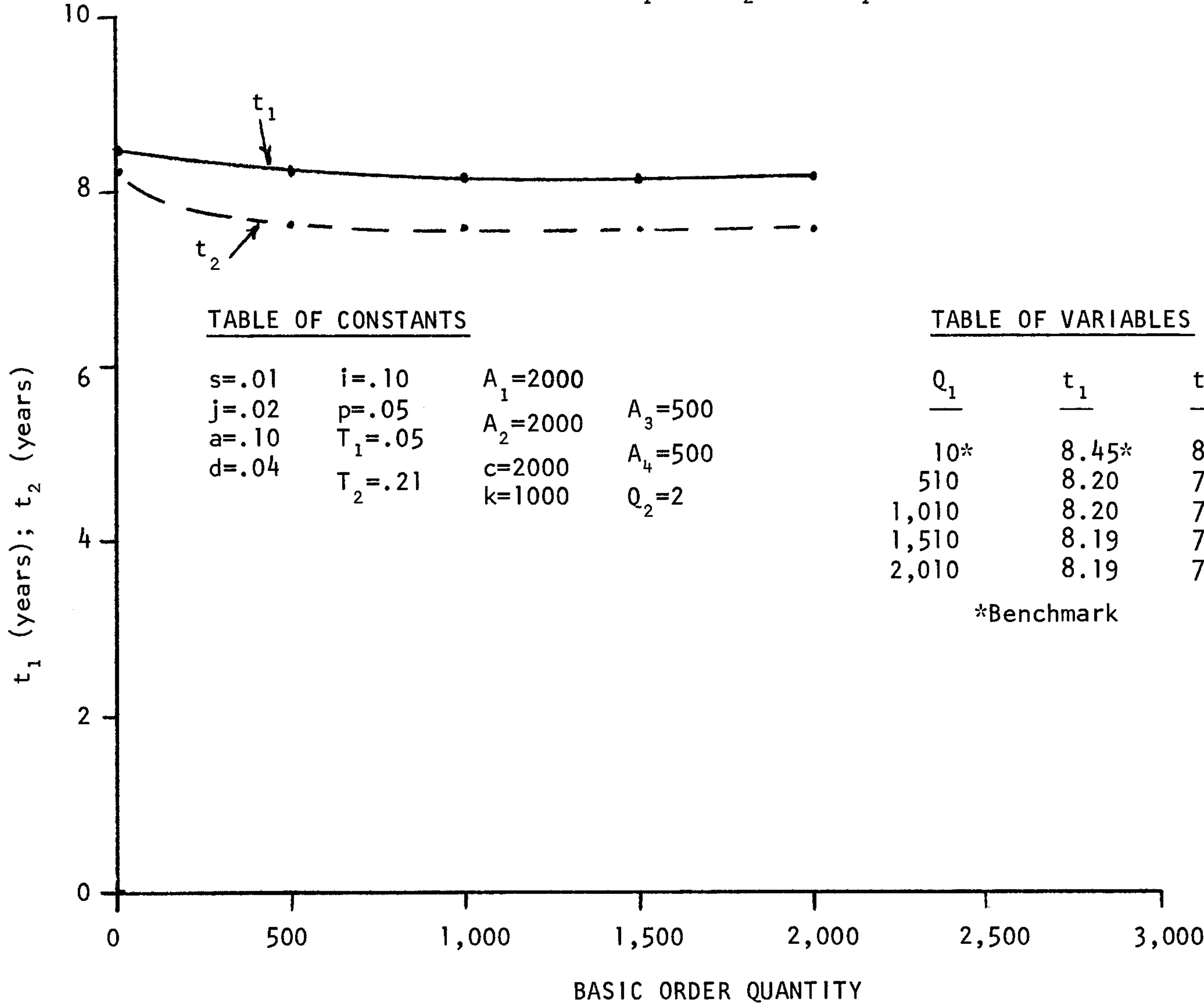

TABLE OF CONSTANTS

$s=.01$	$i=.10$	$A_1=2000$	
$j=.02$	$p=.05$	$A_2=2000$	$A_3=500$
$a=.10$	$T_1=.05$		$A_4=500$
$d=.04$	$T_2=.21$	$c=2000$	
		$k=1000$	$Q_2=2$

TABLE OF VARIABLES

Q_1	t_1	t_2
10*	8.45*	8.25*
510	8.20	7.62
1,010	8.20	7.61
1,510	8.19	7.60
2,010	8.19	7.60

*Benchmark

FIGURE B-11

SENSITIVITY OF t_1 AND t_2 WITH c VARIED

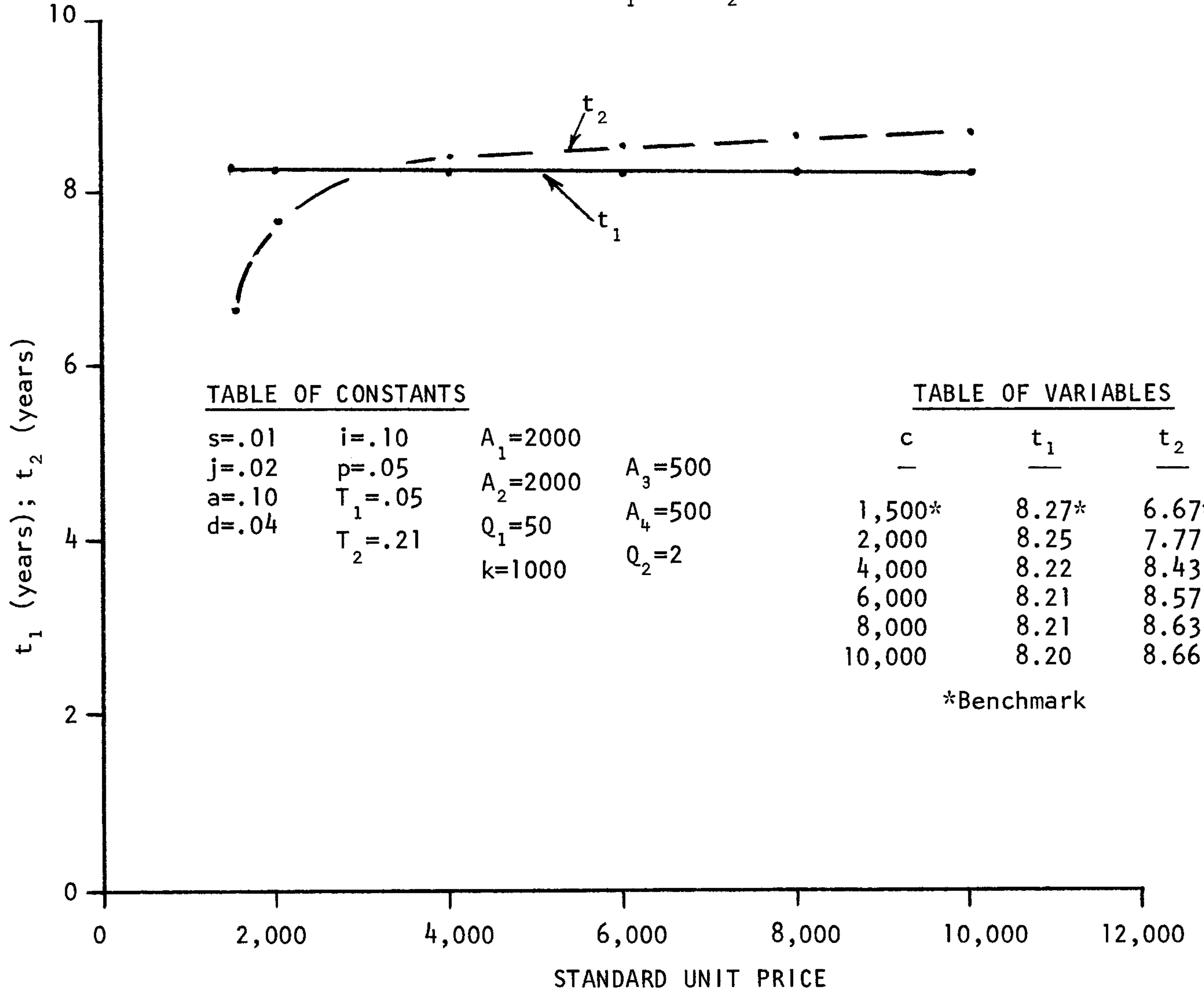

TABLE OF CONSTANTS

s=.01	i=.10	A_1=2000	A_3=500
j=.02	p=.05	A_2=2000	A_4=500
a=.10	T_1=.05	Q_1=50	Q_2=2
d=.04	T_2=.21	k=1000	

TABLE OF VARIABLES

c	t_1	t_2
1,500*	8.27*	6.67*
2,000	8.25	7.77
4,000	8.22	8.43
6,000	8.21	8.57
8,000	8.21	8.63
10,000	8.20	8.66

*Benchmark

FIGURE B-12

SENSITIVITY OF t_2 WITH k VARIED

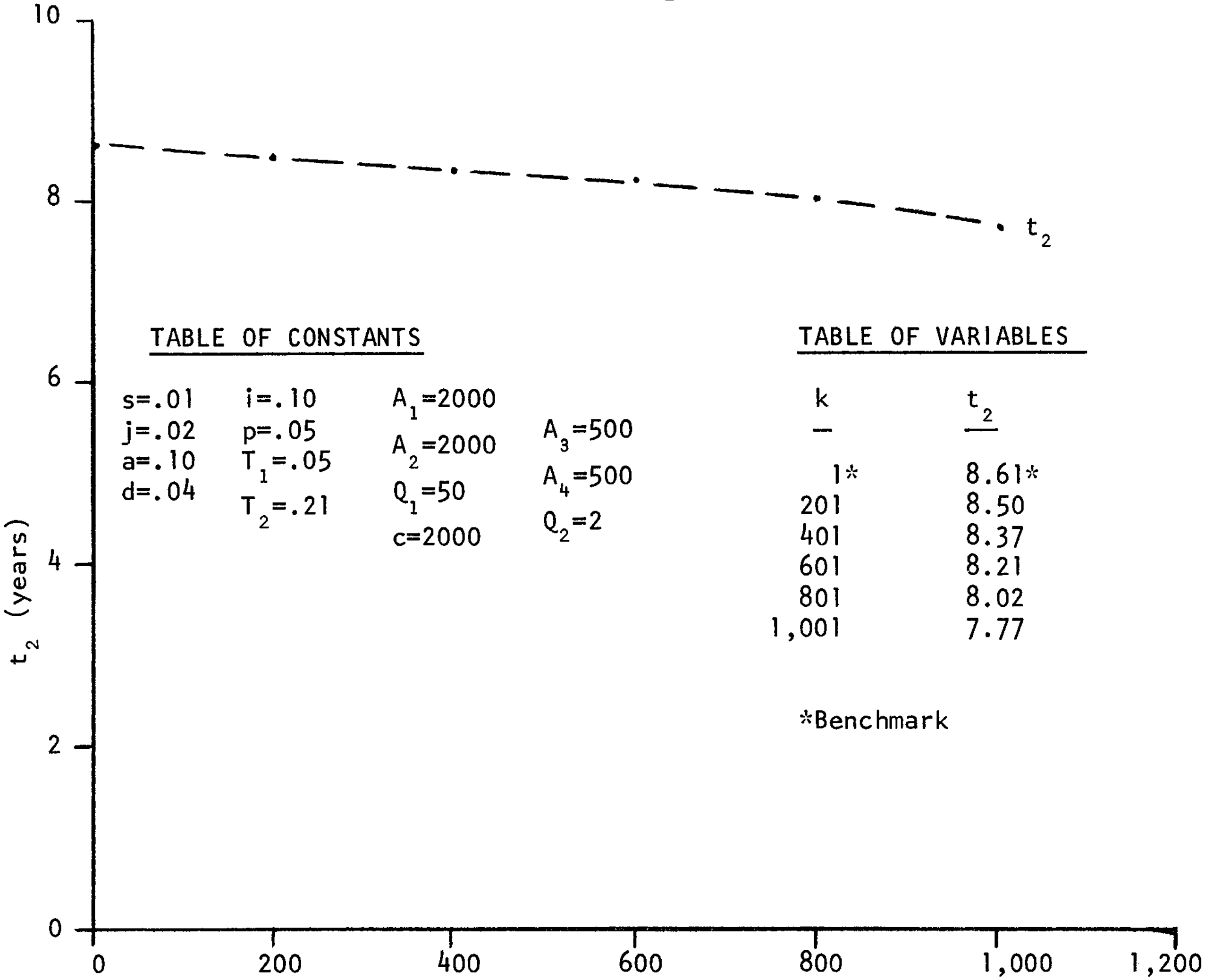

TABLE OF CONSTANTS

s=.01	i=.10	A_1=2000	A_3=500
j=.02	p=.05	A_2=2000	A_4=500
a=.10	T_1=.05	Q_1=50	Q_2=2
d=.04	T_2=.21	c=2000	

TABLE OF VARIABLES

k	t_2
1*	8.61*
201	8.50
401	8.37
601	8.21
801	8.02
1,001	7.77

*Benchmark

FIGURE B-13

SENSITIVITY OF t_2 WITH A_3 VARIED

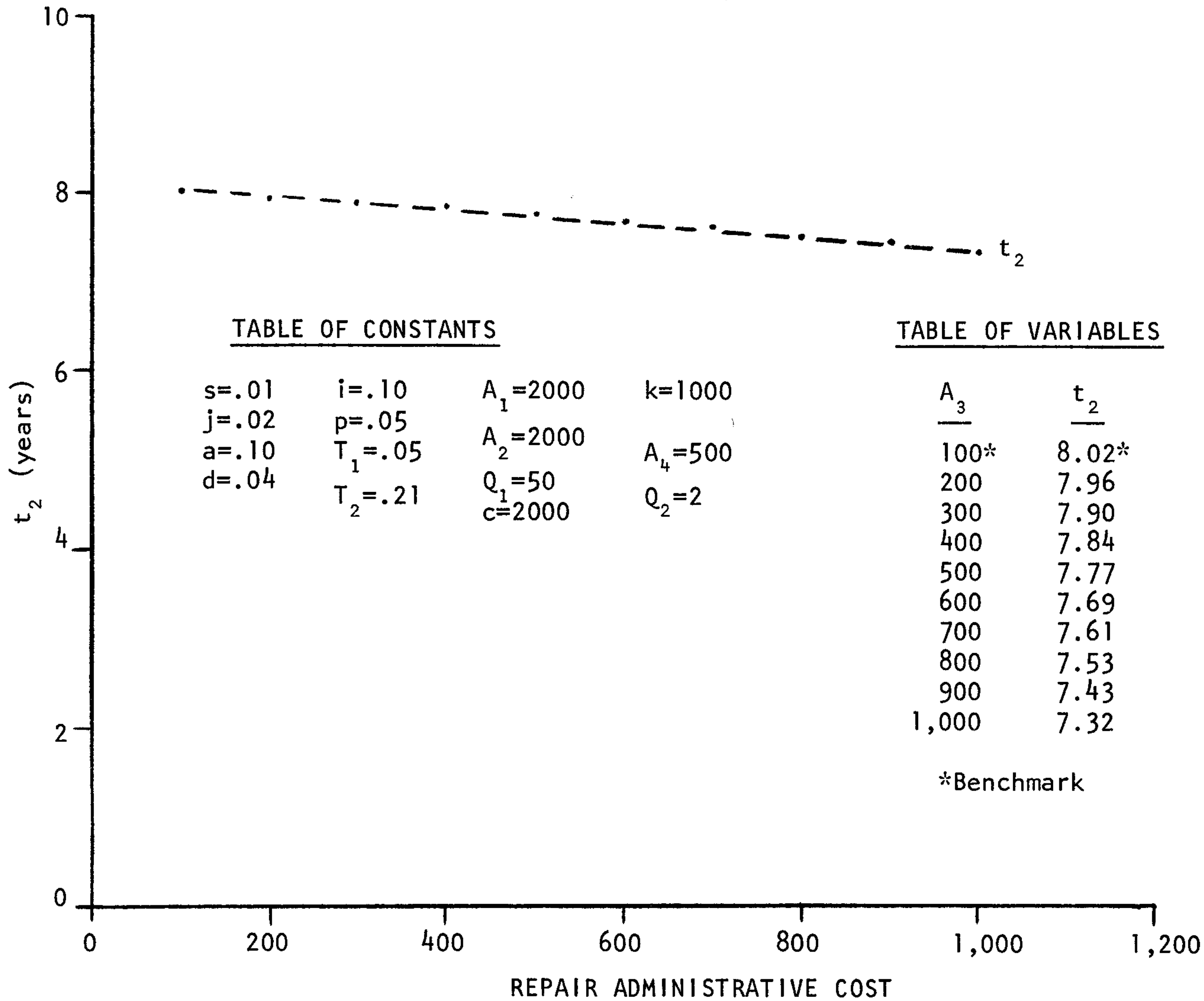

TABLE OF CONSTANTS

s=.01	i=.10	A_1=2000	k=1000
j=.02	p=.05	A_2=2000	A_4=500
a=.10	T_1=.05	Q_1=50	Q_2=2
d=.04	T_2=.21	c=2000	

TABLE OF VARIABLES

A_3	t_2
100*	8.02*
200	7.96
300	7.90
400	7.84
500	7.77
600	7.69
700	7.61
800	7.53
900	7.43
1,000	7.32

*Benchmark

FIGURE B-14

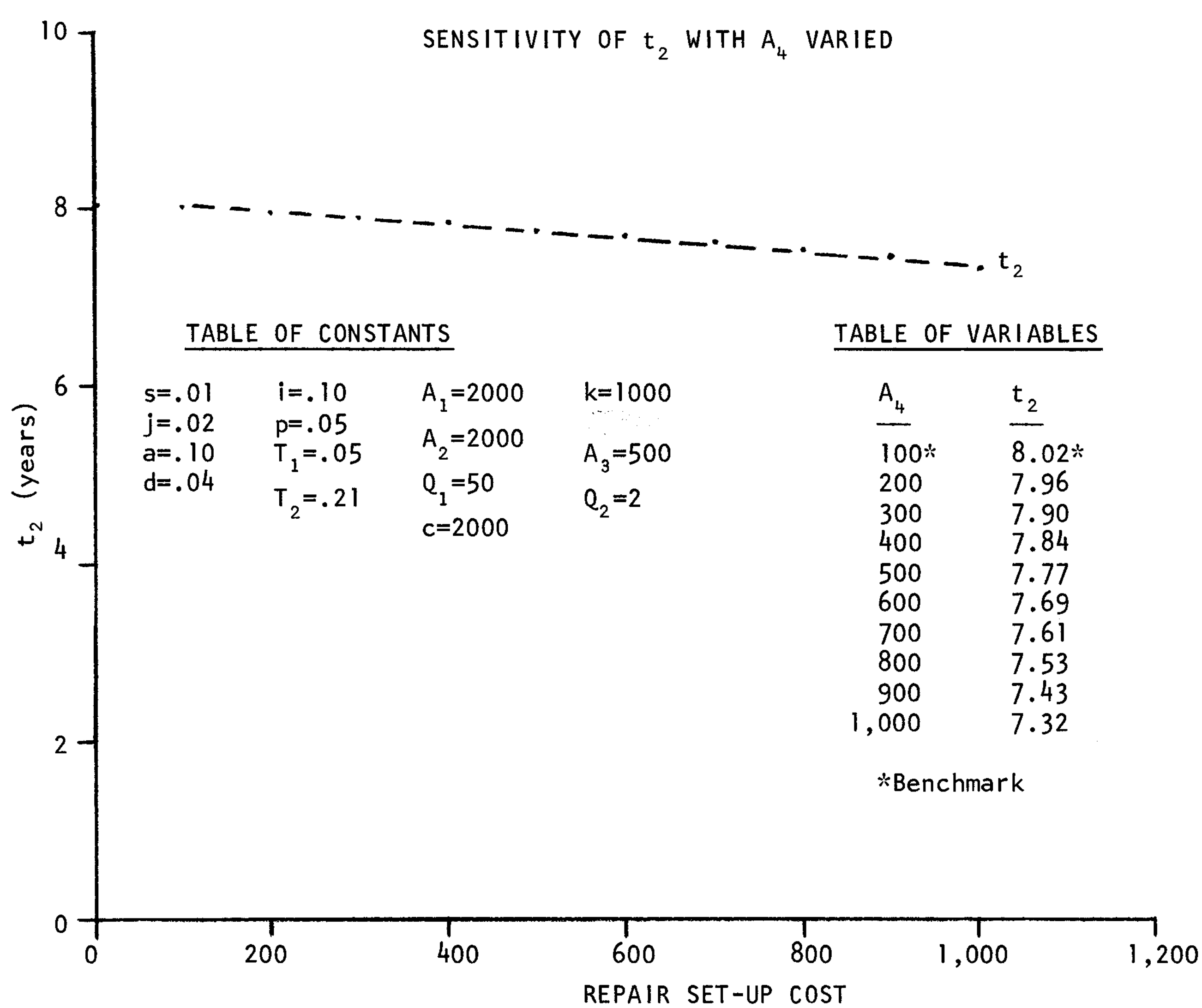

TABLE OF CONSTANTS

s=.01	i=.10	A_1=2000	k=1000
j=.02	p=.05	A_2=2000	A_3=500
a=.10	T_1=.05	Q_1=50	Q_2=2
d=.04	T_2=.21	c=2000	

TABLE OF VARIABLES

A_4	t_2
100*	8.02*
200	7.96
300	7.90
400	7.84
500	7.77
600	7.69
700	7.61
800	7.53
900	7.43
1,000	7.32

*Benchmark

FIGURE B-15

SENSITIVITY OF t_2 WITH Q_2 VARIED

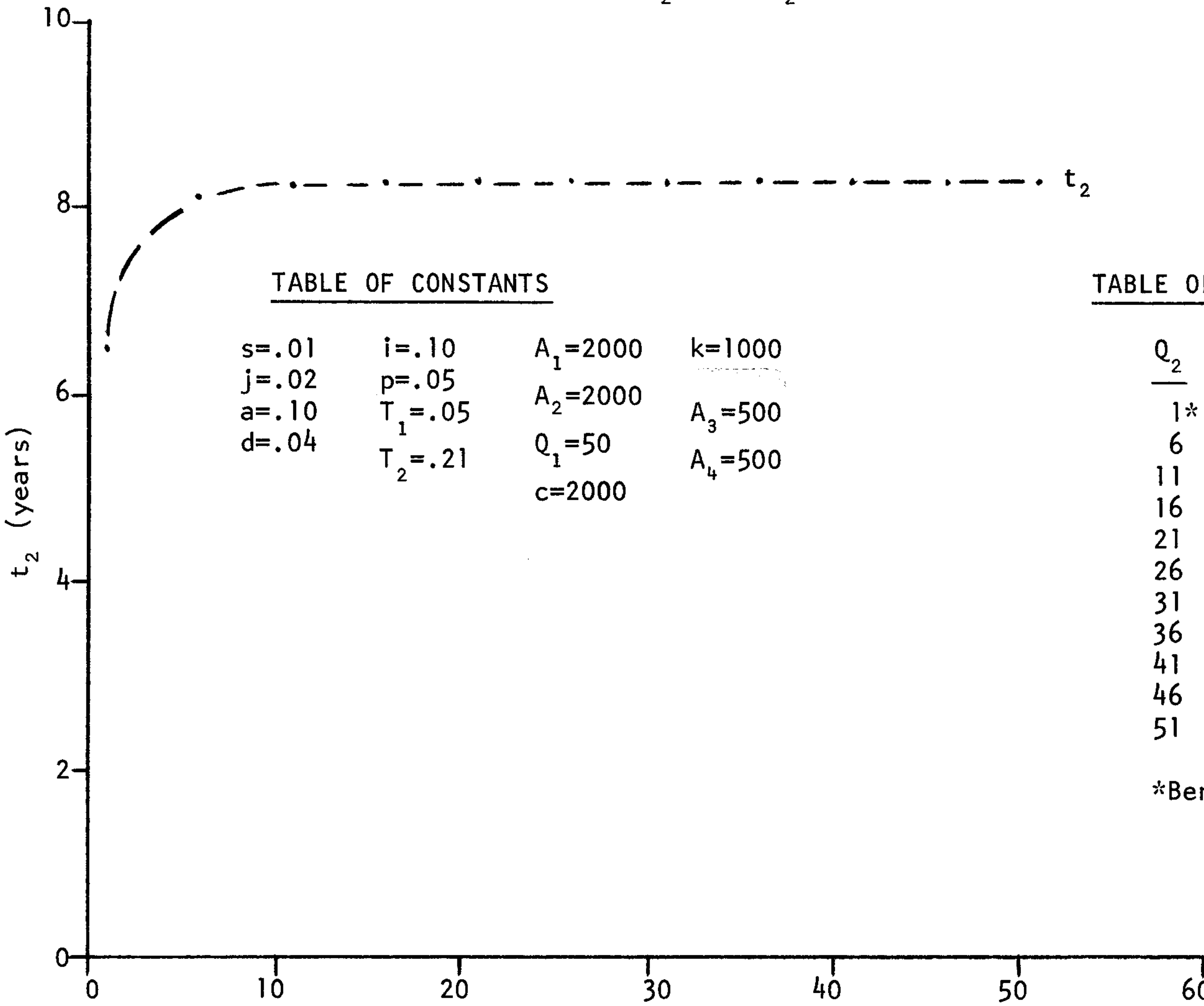

TABLE OF CONSTANTS

s=.01	i=.10	A_1=2000	k=1000
j=.02	p=.05	A_2=2000	A_3=500
a=.10	T_1=.05	Q_1=50	A_4=500
d=.04	T_2=.21	c=2000	

TABLE OF VARIABLES

Q_2	t_2
1*	6.52*
6	8.15
11	8.22
16	8.24
21	8.26
26	8.26
31	8.27
36	8.27
41	8.28
46	8.28
51	8.28

*Benchmark

CONSTRAINED REPAIR QUANTITY

FIGURE B-16

APPENDIX C: DETAILS OF EMPIRICAL ANALYSIS

1. TABLE C-1: Frequency Distribution of Retention Quantities by SPCC Cogs

2. TABLE C-2: Frequency Distribution of Retention Quantity Differences by SPCC Cogs

3. TABLE C-3: Summary Statistics by SPCC Cogs

4. TABLE C-4: Frequency Distribution of Retention Quantities by ASO Cogs

5. TABLE C-5: Frequency Distribution of Retention Quantity Differences by ASO Cogs

6. TABLE C-6: Summary Statistics by ASO Cog

7. TABLE C-7: Frequency Distribution of Retention Quantities by SPCC Cogs

8. TABLE C-8: Frequency Distribution of Retention Quantity Differences by SPCC Cogs

9. TABLE C-9: Summary Statistics by SPCC Cogs

10. TABLE C-10: Frequency Distribution of Retention Quantities by ASO Cogs

11. TABLE C-11: Frequency Distribution of Retention Quantity Differences by ASO Cogs

12. TABLE C-12: Summary Statistics by ASO Cogs

TABLE C-1

FREQUENCY DISTRIBUTION OF RETENTION QUANTITIES BY SPCC COGS

SPCC Items - 1H Cog Consumables; 2H, 4G, 4N Cog Repairables, Priced Out ERR2, Priced Out ERR1, and Priced Out ERR, p=0, j=0

$ FREQUENCY LIMITS	FREQUENCY OF PRICED OUT ERR2				FREQUENCY OF PRICED OUT ERR1				FREQUENCY OF PRICED OUT ERR			
	1H	2H	4G	4N	1H	2H	4G	4N	1H	2H	4G	4N
0	1,169	426	331	684	18,808	359	280	309	20,731	290	219	189
10,000	15,288	1,527	1,107	1,445	6,146	2,388	1,563	2,063	4,293	2,419	1,577	2,122
20,000	3,387	698	430	487	334	666	441	559	297	687	460	592
30,000	1,537	368	234	301	115	306	184	319	93	314	200	332
40,000	988	265	163	192	45	221	125	201	44	223	126	207
50,000	621	182	105	127	25	146	88	138	25	146	90	140
60,000	441	151	100	97	20	117	63	88	15	119	65	89
70,000	323	118	57	87	15	82	50	63	9	83	52	64
80,000	251	94	53	80	2	51	43	57	6	51	44	57
90,000	216	78	47	56	10	45	42	40	7	45	42	41
100,000	146	77	32	57	4	43	30	35	5	43	30	36
110,000	124	57	23	46	4	37	22	29	3	37	23	30
120,000	108	55	23	29	3	40	19	30	4	40	20	31
130,000	96	49	28	35	3	14	20	26	3	14	20	26
140,000	90	36	22	34	1	14	15	25	1	15	16	24
150,000	79	40	19	31	0	19	12	22	0	19	12	22
160,000	71	36	22	19	5	15	8	14	2	15	8	15
170,000	44	32	19	23	0	12	10	13	2	13	10	14
180,000	54	30	13	16	1	12	16	12	0	12	16	12
190,000	48	29	20	9	1	12	6	9	1	12	6	9
200,000	38	26	12	23	1	6	7	13	2	6	7	13
100,000,000	429	407	360	354	5	177	176	166	5	179	177	166
TOTAL	25,548	4,781	3,220	4,231	25,548	4,782	3,220	4,231	25,548	4,782	3,220	4,231

TABLE C-5

FREQUENCY DISTRIBUTION OF RETENTION QUANTITY DIFFERENCES BY ASO COGS

ASO Items - 1R Cog Consumables; 2R Cog Repairables, Frequency of Priced Out ERR minus ERR1; ERR minus ERR2; p=0, j=0

$ FREQUENCY LIMITS	FREQUENCY PRICED OUT ERR MINUS ERR1		FREQUENCY OF PRICED OUT ERR MINUS ERR2	
	1R	2R	1R	2R
-1,000,000	0	1	74	759
- 7,000	584	28	14,174	6,724
- 6,000	137	11	956	192
- 5,000	172	5	1,141	214
- 4,000	257	11	1,346	221
- 3,000	386	24	1,750	279
- 2,000	691	35	2,380	288
- 1,000	1,330	66	3,483	303
0	29,250	11,674	7,338	1,238
1,000	1,964	29	2,149	156
2,000	214	22	212	155
3,000	99	24	102	121
4,000	53	15	57	94
5,000	23	8	16	78
6,000	20	8	19	85
7,000	19	13	12	66
1,000,000	66	80	56	1,052
TOTAL	35,265	12,054	35,265	12,025

TABLE C-2

FREQUENCY DISTRIBUTION OF RETENTION QUANTITY DIFFERENCES BY SPCC COGS

SPCC Items - 1H Cog Consumables; 2H, 4G, 4N Cog Repairables, Frequency of Priced Out ERR minus ERR1; Priced Out ERR minus ERR2; p=0, j=0

$ FREQUENCY LIMITS	FREQUENCY OF PRICED OUT ERR MINUS ERR1				FREQUENCY OF PRICED OUT ERR MINUS ERR2			
	1H	2H	4G	4N	1H	2H	4G	4N
-1,000,000	0	0	0	0	27	44	50	37
-7,000	106	0	4	2	10,409	2,481	1,479	1,681
-6,000	19	0	1	0	824	126	78	67
-5,000	27	0	0	0	919	117	75	69
-4,000	26	0	0	0	995	154	78	60
-3,000	73	2	2	2	1,421	119	70	74
-2,000	127	4	2	1	1,833	114	71	65
-1,000	322	4	3	5	3,003	113	87	88
0	23,079	4,620	3,046	3,969	4,361	674	543	860
1,000	1,495	18	30	40	1,526	139	118	149
2,000	156	28	24	40	168	90	81	136
3,000	52	22	14	28	73	61	57	88
4,000	17	14	12	17	21	57	46	73
5,000	8	12	11	12	14	40	28	56
6,000	14	9	6	19	15	34	28	61
7,000	6	4	6	14	8	23	20	44
1,000,000	21	45	59	82	31	395	306	611
TOTAL	25,548	4,682	3,220	4,231	25,548	4,781	3,215	4,219

TABLE C-3

SUMMARY STATISTICS BY SPCC COGS

SPCC Items - Summary Statistics by Cog comparing Priced Out ERR2, Priced Out ERR1, Priced Out ERR, Priced Out ERR minus ERR1 and Priced Out ERR minus ERR2, p=0, j=0

COG	NR LINE ITEMS	ERR2 TOTAL INVESTMENT	ERR1 TOTAL INVESTMENT	ERR TOTAL INVESTMENT	ERR MINUS ERR1	ERR MINUS ERR2
1H	25,559	$623,145,484.30	$ 29,296,567.09	$ 26,331,783.50	$-2,964,783.59	$-596,813,700.80
2F	309	183,743,362.00	53,638,411.00	53,704,082.00	65,671.00	-130,039,280.00
2H	5,097	620,163,675.50	221,718,921.23	223,716,393.23	1,997,472.00	-396,447,282.27
2J	105	184,519,563.00	81,890,703.00	90,190,703.00	8,300,000.00	- 94,328,860.00
2S	999	559,174,756.00	153,526,576.00	154,002,833.00	476,257.00	-405,171,923.00
2U	345	22,087,311.50	11,401,429.00	11,486,831.00	85,402.00	- 10,600,480.50
2Z	439	135,855,494.90	52,253,494.40	52,550,742.70	297,248.30	- 83,304,752.20
4A	361	45,955,999.50	18,899,805.20	19,166,758.20	266,953.00	- 26,789,241.30
4G	3,381	582,313,314.93	219,019,949.96	220,903,632.46	1,883,682.50	-361,409,682.47
4N	4,356	390,970,661.45	202,594,004.48	204,872,159.28	2,278,154.80	-186,098,502.17
40	2	57,250.00	10,100.00	10,100.00	0	- 47,150.00
4U	509	385,239,130.20	129,563,143.50	130,255,015.00	691,871.50	-254,984,115.20
6A	947	18,051,071.88	12,531,246.68	14,306,954.10	1,775,707.42	- 3,744,117.78
6E	143	14,846,354.50	5,438,266.00	5,563,586.00	125,320.00	- 9,282,768.50
6G	455	41,258,064.24	13,310,852.82	12,847,593.32	- 463,259.50	- 28,410,470.92
6H	1,174	41,040,355.66	22,660,144.04	23,925,831.88	1,265,687.84	- 17,114,523.78
6M	12	261,112.00	260,336.00	260,040.00	- 296.00	- 1,072.00
60	41	734,731.00	233,118.00	243,582.00	10,464.00	- 491,149.00
6U	365	9,384,800.30	7,467,234.44	7,820,606.94	353,372.50	- 1,564,193.36
6X	208	3,982,039.56	1,060,569.72	1,064,269.72	3,700.00	- 2,917,769.84
8H	180	12,418,342.10	11,226,855.80	11,249,252.30	22,396.50	- 1,169,089.80

TABLE C-4

FREQUENCY DISTRIBUTION OF RETENTION QUANTITIES BY ASO COGS

ASO Items - 1R Cog Consumables, 2R Cog Repairables, Priced Out ERR2, Priced Out ERR1, and Priced Out ERR; p=0, j=0

$ FREQUENCY LIMITS	FREQUENCY OF PRICED OUT ERR2		FREQUENCY OF PRICED OUT ERR1		FREQUENCY OF PRICED OUT ERR	
	1R	2R	1R	2R	1R	2R
0	3,120	1,244	22,737	426	29,517	313
10,000	19,612	1,931	11,590	3,219	5,130	3,268
20,000	4,332	1,193	553	1,543	336	1,576
30,000	2,220	815	172	903	122	906
40,000	1,321	569	81	592	70	601
50,000	889	451	43	495	19	496
60,000	632	379	20	404	17	407
70,000	454	290	17	302	16	307
80,000	346	280	12	249	6	249
90,000	319	219	6	234	6	235
100,000	212	179	5	219	6	221
110,000	201	171	6	178	3	176
120,000	169	160	6	145	3	146
130,000	143	138	2	139	0	140
140,000	105	159	2	104	4	102
150,000	95	117	1	100	0	102
160,000	89	100	3	102	2	104
170,000	82	105	1	95	0	98
180,000	62	104	0	79	0	77
190,000	70	87	0	77	0	76
200,000	51	83	0	71	1	72
100,000,000	741	3,278	8	2,379	7	2,383
TOTAL	35,265	12,052	35,265	12,055	35,265	12,055

TABLE C-6

SUMMARY STATISTICS BY ASO COG

ASO Items - Summary Statistics by Cog comparing Priced Out ERR2, Priced Out ERR1, Priced Out ERR, Priced Out ERR minus ERR1, Priced Out ERR minus ERR2; p=0, j=0

COG	NR LINE ITEMS	ERR2 TOTAL INVESTMENT	ERR1 TOTAL INVESTMENT	ERR TOTAL INVESTMENT	ERR MINUS ERR1	ERR MINUS ERR2
1R	35,383	$ 995,954,477.37	$ 52,910,599.14	$ 36,351,520.73	$-16,559,078.41	$- 959,602,956.64
2R	12,100	7,960,561,061.31	3,874,100,386.98	3,887,246,198.78	13,145,811.80	-4,073,314,862.53
2V	15	23,725,780.00	10,205,453.00	10,205,453.00	0	- 13,520,327.00
2W	56	8,287,803.00	4,199,670.00	4,203,912.00	4,242.00	- 4,083,891.00
4R	79	14,921,578.57	5,512,491.02	5,512,491.02	0	- 9,409,087.55
4Z	22	36,075,681.00	22,354,728.00	22,376,742.00	0	- 13,698,939.00
5R	1,336	41,340,781.33	1,678,706.77	1,580,726.99	- 97,979.78	- 39,760,054.34
6R	1,209	131,488,275.50	67,872,141.50	71,148,576.00	3,276,434.50	- 60,339,699.50
8N	228	3,847,724.72	4,601,816.02	4,685,871.02	84,055.00	838,146.30
8R	90	823,167,437.00	261,585,014.00	261,662,803.00	77,789.00	- 561,504,634.00

TABLE C-7

FREQUENCY DISTRIBUTION OF RETENTION QUANTITIES BY SPCC COGS

SPCC Items - 1H Cog Consumables, 2H, 4G, 4N Cog Repairables, Priced Out ERR2, Priced Out ERR1, and Priced Out ERR; p=.05, j=.02

$ FREQUENCY LIMITS	FREQUENCY OF PRICED OUT ERR2				FREQUENCY OF PRICED OUT ERR1				FREQUENCY OF PRICED OUT ERR			
	1H	2H	4G	4N	1H	2H	4G	4N	1H	2H	4G	4N
0	1,663	438	334	699	18,842	364	280	312	20,731	290	219	189
10,000	16,134	1,668	1,187	1,543	6,159	2,394	1,567	2,079	4,293	2,419	1,577	2,122
20,000	3,032	718	458	490	312	656	441	546	297	687	460	592
30,000	1,380	382	242	309	105	309	181	318	93	314	200	332
40,000	843	239	141	188	35	219	125	197	44	223	126	207
50,000	503	196	108	138	29	145	89	138	25	146	90	140
60,000	360	126	92	94	19	117	63	87	15	119	65	89
70,000	258	115	56	94	9	81	49	64	9	83	52	64
80,000	213	105	42	64	7	52	43	57	6	51	44	57
90,000	149	75	37	53	7	46	42	39	7	45	42	41
100,000	137	64	33	47	6	44	32	35	5	43	30	36
110,000	112	58	25	41	3	34	22	31	3	37	23	30
120,000	82	47	29	31	0	40	17	30	4	40	20	31
130,000	89	41	25	32	3	14	19	24	3	14	20	26
140,000	62	27	16	25	1	14	15	25	1	15	16	24
150,000	61	38	23	20	0	19	12	22	0	19	12	22
160,000	40	28	18	20	3	15	10	14	2	15	8	15
170,000	35	28	25	15	0	12	11	13	2	13	10	14
180,000	25	23	15	15	1	12	14	12	0	12	16	12
190,000	28	23	18	15	1	12	5	9	1	12	6	9
200,000	23	27	15	15	1	6	7	13	2	6	7	13
100,000,000	319	315	281	283	5	177	176	166	5	179	177	166
TOTAL	25,548	4,781	3,220	4,231	25,548	4,782	3,220	4,231	25,548	4,782	3,220	4,231

TABLE C-8

FREQUENCY DISTRIBUTION OF RETENTION QUANTITY DIFFERENCES BY SPCC COGS

SPCC Items - 1H Cog Consumables, 2H, 4G, 4N Cog Repairables, Frequency of Priced Out ERR minus ERR1, Priced Out ERR minus ERR2; p=.05, j=.02

$ FREQUENCY LIMITS	FREQUENCY OF PRICED OUT ERR MINUS ERR1				FREQUENCY OF PRICED OUT ERR MINUS ERR2			
	1H	2H	4G	4N	1H	2H	4G	4N
-1,000,000	0	0	2	0	22	26	32	21
- 7,000	104	0	0	1	9,053	2,258	1,378	1,540
- 6,000	19	0	0	0	736	152	65	88
- 5,000	24	0	0	0	885	132	91	82
- 4,000	27	0	0	0	1,154	185	100	77
- 3,000	70	0	0	0	1,240	180	100	96
- 2,000	120	1	0	0	1,807	135	76	83
- 1,000	307	0	0	0	2,938	126	92	92
0	22,725	4,474	2,919	3,775	5,597	576	451	701
1,000	1,455	112	117	155	1,331	237	209	266
2,000	322	55	42	79	348	120	101	176
3,000	133	36	22	47	143	77	65	107
4,000	74	17	16	22	78	62	50	77
5,000	49	16	14	22	58	43	32	68
6,000	29	10	6	26	44	36	28	71
7,000	17	6	7	14	22	25	21	44
1,000,000	73	55	75	90	92	411	324	630
TOTAL	25,548	4,782	3,220	4,231	25,548	4,781	3,215	4,219

TABLE C-9

SUMMARY STATISTICS BY SPCC COGS

SPCC Items - Summary Statistics by Cog comparing Priced Out ERR2, Priced Out ERR1, Priced Out ERR, ERR minus ERR1, ERR minus ERR2, p=.05, j=.02

COG	NR LINE ITEMS	ERR2 TOTAL INVESTMENT	ERR1 TOTAL INVESTMENT	ERR TOTAL INVESTMENT	ERR MINUS ERR1	ERR MINUS ERR2
1H	25,559	$492,716,879.89	$ 27,007,837.39	$ 26,331,783.50	$ 676,053.89	$-466,385,096.39
2F	309	148,383,693.00	53,551.739.00	53,704,082.00	152,343.00	- 94,679,611.00
2H	5,097	483,412,900.32	221,388,528.73	223,716,393.23	2,327,864.50	-259,696,507.09
2J	105	139,007,802.00	81,201,581.00	90,190,703.00	8,989,122.00	- 48,817,099.00
2S	999	452,381,703.00	151,409,803.00	154,002,833.00	2,593,030.00	-298,378,870.00
2U	345	17,791,349.40	11,375,986.90	11,486,831.00	110,844.10	- 6,304,518.40
2Z	439	111,171,618.40	51,749,696.40	52,550,742.70	801,046.30	- 58,620,875.70
4A	361	35,271,827.90	18,734,415.20	19,166,758.20	432,343.00	- 16,105,069.70
4G	3,381	443,427,176.82	218,491,103.46	220,903,632.46	2,412,529.00	-222,523,544.36
4N	4,356	299,848,461.45	202,022,046.48	204,872,159.28	2,850,112.80	- 94,976,302.17
40	2	46,390.00	10,100.00	10,100.00	0	- 36,290.00
4U	509	295,634,873.60	129,508,457.00	130,255,015.00	746,558.00	-165,379,858.60
6A	947	16,245,646.68	12,489,227.68	14,306,954.10	1,817,726.42	- 1,938,692.58
6E	143	11,491,561.50	5,338,588.00	5,563,586.00	224,998.00	- 5,927,975.50
6G	455	32,314,417.84	13,216,165.32	12,847,593.32	-368,572.00	- 19,466,824.52
6H	1,174	35,490,834.92	22,631,933.64	23,925,831.88	1,293,898.24	- 11,565,003.04
6M	12	248,456.00	258,264.00	260,040.00	1,776.00	11,584.00
60	41	589,688.00	233,118.00	243,582.00	10,464.00	- 346,106.00
6U	365	7,479,971.80	7,285,003.94	7,820,606.94	535,603.00	340,635.14
6X	208	3,283,120.96	1,057,967.72	1,064,269.72	6,302.00	- 2,218,851.24
8H	180	11,012,846.30	11,190,407.80	11,249,252.30	58,844.50	236,406.00

TABLE C-10

FREQUENCY DISTRIBUTION OF RETENTION QUANTITIES BY ASO COGS

ASO Items - 1R Cog Consumables; 2R Cog Repairables, Priced Out ERR2, Priced Out ERR1 and Priced Out ERR; p=.05, j=.02

$ FREQUENCY LIMITS	FREQUENCY OF PRICED OUT ERR2		FREQUENCY OF PRICED OUT ERR1		FREQUENCY OF PRICED OUT ERR	
	1R	2R	1R	2R	1R	2R
0	4,501	1,449	22,792	453	29,517	313
10,000	20,484	2,211	11,621	3,243	5,130	3,268
20,000	3,895	1,313	511	1,529	336	1,576
30,000	1,877	809	161	886	122	906
40,000	1,087	574	72	594	70	601
50,000	707	470	33	491	19	496
60,000	480	369	16	409	17	407
70,000	367	291	16	306	16	307
80,000	280	228	9	240	6	249
90,000	222	240	6	233	6	235
100,000	163	181	3	218	6	221
110,000	119	173	5	175	3	176
120,000	131	161	6	144	3	146
130,000	106	134	1	138	0	140
140,000	73	115	1	102	4	102
150,000	67	111	1	102	0	102
160,000	56	95	2	99	2	104
170,000	49	103	1	95	0	98
180,000	44	89	0	76	0	77
190,000	41	86	0	77	0	76
200,000	40	73	0	72	1	72
100,000,000	476	2,780	8	2,373	7	2,383
TOTAL	35,265	12,055	35,265	12,055	35,265	12,055

TABLE C-11

FREQUENCY DISTRIBUTION OF RETENTION QUANTITY DIFFERENCES BY ASO COGS

ASO Items - 1R Cog Consumables; 2R Cog Repairables; Frequency of Priced Out ERR minus ERR1, Priced Out ERR minus ERR2; p=.05, j=.02

$ FREQUENCY LIMITS	FREQUENCY OF PRICED OUT ERR MINUS ERR1		FREQUENCY OF PRICED OUT ERR MINUS ERR2	
	1R	2R	1R	2R
-1,000,000	0	1	48	423
- 7,000	566	1	11,887	5,551
- 6,000	136	2	944	217
- 5,000	167	0	1,074	251
- 4,000	247	1	1,301	278
- 3,000	370	7	1,630	323
- 2,000	669	7	2,300	363
- 1,000	1,289	10	3,471	337
0	28,780	11,091	9,631	881
1,000	1,961	343	1,859	506
2,000	478	169	487	363
3,000	185	104	193	231
4,000	102	46	91	162
5,000	72	31	68	122
6,000	42	23	41	126
7,000	48	28	62	99
1,000,000	153	190	178	1,779
TOTAL	35,265	12,054	35,265	12,012

TABLE C-12

SUMMARY STATISTICS BY ASO COGS

ASO Items - Summary Statistics by Cog comparing Priced Out ERR2, Priced Out ERR1, Priced Out ERR minus ERR1, Priced Out ERR minus ERR2; p=.05, j=.02

COG	NR LINE ITEMS	ERR2 TOTAL INVESTMENT	ERR1 TOTAL INVESTMENT	ERR TOTAL INVESTMENT	ERR MINUS ERR1	ERR MINUS ERR2
1R	35,383	$ 720,844,584.45	$ 49,401,073.26	$ 36,351,520.73	$-13,049,552.53	$- 684,493,063.72
2R	12,100	5,769,976,343.78	3,869,472,055.78	3,887,246,198.78	17,774,143.00	-1,882,730,145.00
2V	15	18,695,672.00	10,204,732.00	10,205,453.00	721.00	- 8,490,219.00
2W	56	6,796,995.00	4,070,193.00	4,203,912.00	133,719.00	- 2,593,083.00
4R	79	11,521,821.06	5,511,528.02	5,512,491.02	963.00	- 6,009,330.04
4Z	22	27,653,677.00	22,351,232.00	22,376,742.00	25,510.00	- 5,276,935.00
5R	1,336	33,731,692.08	1,611,159.07	1,580,726.99	-30,432.08	- 32,150,965.09
6R	1,209	110,538,279.55	67,458,202.30	71,148,576.00	3,690,373.70	- 39,389,703.55
8N	228	3,688,584.72	4,601,816.02	4,685,871.02	84,055.00	997,286.30
8R	90	597,679,508.00	261,455,874.00	261,662,803.00	206,929.00	- 336,016,705.00

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13. ABSTRACT

This study evaluates alternative UICP Navy Economic Retention models. The current Navy Economic Retention Model was developed in 1965 for consumables only and was restricted in precision by computer constraints and simplifying assumptions. A replacement model is proposed that applies to RFI (Ready-for-Issue) consumable and repairable assets, as well as NRFI (Not-Ready-For-Issue) repairable assets. The proposed model represents an improved mathematical formulation that takes advantage of current ADP (Automatic Data Processing) capabilities and, thus, eliminates many simplifying assumptions of the current model. The proposed model, under current constraints, computed a lower economic retention requirement for the total of all Navy items. Holding costs were expected to decrease between $165K and $331K. However, eimplementation of the proposed model based solely on economic criteria would increase the economic retention quantities and increase holding costs between $47,387K and $83,451K.

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